1. **Vocabulary** An equation with one or more absolute-value expressions is called an ________.

Simplify.

2. \(|8 - 15|\)  

3. \(|-3 + 9|\)

**Solve.**

4. \(|x - 4| = 7\)

5. \(|x + 7| = 2\)

**New Concepts** An absolute-value inequality is an inequality with at least one absolute-value expression. The solution to an absolute-value inequality can be written as a compound inequality.

The inequality \(|x| < 6\) describes all real numbers whose distance from 0 is less than 6 units. The solutions are all real numbers between \(-6\) and 6. The solution can be written \(-6 < x < 6\) or as the compound inequality \(x > -6\) AND \(x < 6\).

The inequality \(|x| > 6\) describes all real numbers whose distance from 0 is greater than 6 units. The solutions are all real numbers less than \(-6\) or greater than 6. The solution can be written as the compound inequality \(x < -6\) OR \(x > 6\).

**Example 1** Solving Absolute-Value Inequalities by Graphing

Solve each inequality by graphing.

a. \(|x| < 4\)

**SOLUTION**

If the absolute value of \(x\) is less than 4, then \(x\) is less than 4 units from zero on a number line.

The graph shows \(x < 4\) AND \(x > -4\). This can also be written \(-4 < x < 4\).

b. \(|x| > 7\)

**SOLUTION**

If the absolute value of \(x\) is greater than 7, then \(x\) is more than 7 units from zero on a number line.

The graph shows \(x > 7\) OR \(x < -7\).
Example 2  Isolating the Absolute Value to Solve

Solve and graph each inequality.

a. \(|x| + 7.4 \leq 9.8\)

**SOLUTION**

Begin by isolating the absolute value.

\(|x| + 7.4 \leq 9.8\)

\(-7.4 \quad -7.4\)  Subtraction Property of Inequality

\(|x| \leq 2.4\)  Simplify.

Since the absolute value of \(x\) is less than or equal to 2.4, it is 2.4 units or less from zero.

The solution can be written \(x \geq -2.4\) AND \(x \leq 2.4\) or \(-2.4 \leq x \leq 2.4\).

b. \(\frac{|x|}{4} > 2\)

**SOLUTION**

Begin by isolating the absolute value.

\(\frac{|x|}{4} > 2\)

\(4 \cdot \frac{|x|}{4} > 2 \cdot 4\)  Multiplication Property of Inequality

\(|x| > 8\)  Simplify.

The absolute value of \(x\) is greater than 8, so it is more than 8 units from zero.

The solution is \(x > 8\) OR \(x < -8\).

c. \(-2|x| < -6\)

**SOLUTION**

Begin by isolating the absolute value.

\(-2|x| < -6\)

\(\frac{-2|x|}{-2} > \frac{-6}{-2}\)  Division Property of Inequality

\(|x| > 3\)  Simplify.

Since the absolute value of \(x\) is greater than 3, it is more than 3 units from zero.

The solution is \(x > 3\) OR \(x < -3\).
Some absolute-value inequalities have variable expressions inside the absolute-value symbols. The expression inside the absolute-value symbols can be positive or negative.

The inequality $|x + 1| < 3$ represents all numbers whose distance from $-1$ is less than $3$.

The inequality $|x + 1| > 3$ represents all numbers whose distance from $-1$ is greater than $3$.

### Rules for Solving Absolute-Value Inequalities

<table>
<thead>
<tr>
<th>Inequality Form</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>K</td>
</tr>
<tr>
<td>$</td>
<td>K</td>
</tr>
</tbody>
</table>

Similar rules are true for $|K| \leq a$ or $|K| \geq a$.

### Example 3 Solving Inequalities with Operations Inside Absolute-Value Symbols

Solve each inequality. Then graph the solution.

**a.** $|x - 5| \leq 3$

**SOLUTION**

Use the rules for solving absolute-value inequalities to write a compound inequality.

$|x - 5| \leq 3$

$x - 5 \geq -3$ AND $x - 5 \leq 3$ Write the compound inequality.

$\begin{align*}
+5 & \quad +5 \\
\phantom{-}x & \geq 2 \quad \text{AND} \quad x \leq 8
\end{align*}$

Addition Property of Inequality Simplify.

Now graph the inequality.

**b.** $|x + 7| > 3$

**SOLUTION**

Use the rules for solving absolute-value inequalities to write a compound inequality.

$|x + 7| > 3$

$x + 7 < -3$ OR $x + 7 > 3$ Write the compound inequality.

$\begin{align*}
-7 & \quad -7 \\
\phantom{+}x & < -10 \quad \text{OR} \quad x > -4
\end{align*}$

Subtraction Property of Inequality Simplify.

Now graph the inequality.
Example 4  Solving Special Cases

Solve each inequality.

a. \(|x| + 6 \leq 4\)

**SOLUTION**

\(|x| + 6 \leq 4\)

\(|x| \leq -2\) Subtract 6 from both sides.

This inequality states that a number’s distance from 0 is less than or equal to \(-2\). No distance can be negative. Therefore, there are no solutions to this inequality. The solution is identified as \(\emptyset\) or \(\varnothing\), the empty set.

b. \(|x| + 6 > 1\)

**SOLUTION**

\(|x| + 6 > 1\)

\(|x| > -5\) Subtract 6 from both sides.

This inequality states that a number’s distance from 0 is greater than \(-5\). Since all distances (and absolute values) are positive, all numbers on the number line are solutions. The solution is identified as \(\mathbb{R}\), the set of all real numbers. It means that the inequality is an identity; it works for all real numbers.

Example 5  Application: Polling

A poll finds that candidate Garcia is favored by 46% of the voters surveyed and Jackson is favored by 44%. The poll has an accuracy of plus or minus 3%.

a. Write an absolute-value inequality to show the true percentage of voters for Garcia.

**SOLUTION**

Let the true percentage of voters for Garcia be \(g\).

For \(g\) to be within 3%, the distance from \(g\) to 46 must be less than or equal to 3. The distance is represented by the absolute value of their difference.

\(|g - 46| \leq 3\)

b. Solve the inequality to find the range for the true percentage of voters who support Garcia.

**SOLUTION**

\(|g - 46| \leq 3\)

\(-3 \leq g - 46 \leq 3\) Write the inequality without an absolute value.

\(43 \leq g \leq 49\) Add 46 to all 3 parts of the inequality.

The range is between 43% and 49%.
Lesson Practice

a. Solve and graph the inequality $|x| < 12.$

b. Solve and graph the inequality $|x| > 19.$

c. Solve and graph the inequality $|x| + 2.8 \leq 10.4.$

d. Solve and graph the inequality $\frac{|x|}{5} < -1.$

e. Solve and graph the inequality $|x - 10| \leq 12.$

f. Solve and graph the inequality $|x + 12| > 18.$

g. Solve the inequality $|x| + 21 \leq 14.$

h. Solve the inequality $|x| + 33 > 24.$

(Ex 5) A machine part must be $15 \pm 0.2$ cm in diameter.

i. Write an inequality to show the range of acceptable diameters.

j. Solve the inequality to find the actual range for the diameters.

Practice  Distributed and Integrated

1. Find the axis of symmetry for the graph of the equation $y = -\frac{1}{2}x^2 + x - 3.$

Add or subtract.

2. $\frac{6rs}{r^2s^2} + \frac{18r}{r^2s^2}$

3. $\frac{b}{2b + 1} - \frac{6}{b - 4}$

Factor.

4. $-4y^4 + 8y^3 + 5y^2 - 10y$

5. $3a^2 - 27$

Evaluate.

6. $4x^2 + 6x - 4$

7. $9x^2 - 2x + 32$

*8. Solve and graph the inequality $|x| < 96.$

*9. Write  Explain what $|x| \geq 54$ means on a number line.

*10. Justify  When solving an absolute-value inequality, a student gets $|x| \geq -5.$ Justify that any value for $x$ makes this inequality true.

*11. Multiple Choice  Which inequality is represented by the graph?

- A $|x| < 9$
- B $|x| > 9$
- C $|x| \leq 9$
- D $|x| < -9$

*12. Track  A runner finishes a sprint in 8.54 seconds. The timer’s accuracy is plus or minus 0.3 seconds. Solve and graph the inequality $|t - 8.54| \leq 0.3.$
*13. **Error Analysis**  Two students simplified \( \frac{2c}{c - 6} + \frac{12}{6 - c} \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2c - 12}{c - 6} )</td>
<td>( \frac{2c + 12}{c - 6} )</td>
</tr>
<tr>
<td>( \frac{2(c - 6)}{c - 6} = 2 )</td>
<td>( \frac{2(c + 6)}{c - 6} = -2 )</td>
</tr>
</tbody>
</table>

*14. **Multi-Step**  A farmer has a rectangular plot of land with an area of \( x^2 + 22x + 72 \) square meters. He sets aside \( x^2 \) square meters for grazing and \( 2x - 8 \) square meters for a chicken coop.

a. Write a simplified expression for the total fraction of the field the farmer has set aside.

b. **Estimate**  About what percent of the field has the farmer set aside if \( x = 30 \)?

*15. **Geometry**  Write a simplified expression for the total fraction of the larger rectangle that the triangle and smaller rectangle cover.

16. Find the product of \( (\sqrt{4} - 6)^2 \).

17. **Analyze**  Why is it necessary to understand factoring when dealing with rational expressions?

18. **Multi-Step**  The base of triangle \( ABC \) is \( x^2 + y \). The height is \( \frac{4x + 2xy}{x^2 + xy} \). What is the area of triangle \( ABC \)?

a. Multiply the base of the triangle by its height.

b. Multiply the product from part a by \( \frac{1}{2} \).

*19. **Error Analysis**  Two students tried to find the axis of symmetry for the equation \( y = 8x + 2x^3 \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = \frac{-b}{2a} = \frac{-2}{2(8)} = \frac{-2}{16} = -\frac{1}{8} )</td>
<td>( x = \frac{-8}{2(2)} = \frac{-8}{4} = -2 )</td>
</tr>
</tbody>
</table>

*20. **Space**  If it were possible to play ball on Jupiter, the function \( y = -13x^2 + 39x \) would approximate the height of a ball kicked straight up at a velocity of 39 meters per second, where \( x \) is time in seconds. Find the maximum height the ball reaches and the time it takes the ball to reach that height. (Hint: Find the time the ball reaches its maximum height first.)

21. **Measurement**  The coordinates of two landmarks on a city map are \( A(5, 3) \) and \( B(7, 10) \). Each grid line represents 0.05 miles. Find the distance between landmarks \( A \) and \( B \).
22. **Archeology** Archeologists use coordinate grids to record locations of artifacts. Jonah recorded that he found one old coin at \((41, 37)\), and a second old coin at \((5, 2)\). Each unit on his grid represents 0.25 feet. How far apart were the coins? Round your answer to the nearest tenth of a foot.

23. Find the length \(t\) to the nearest tenth.

24. **House Painting** A house painter leans a 34-foot ladder against a house with the bottom of the ladder 7 feet from the base of the house. Will the top of the ladder touch the house above or below a windowsill that is 33 feet off the ground?

25. Graph the function \(y = 4x^2\).

26. **Generalize** What is the factored form of \(a^{2m} + 2a^m b^n + b^{2n}\)?

27. **Shopping** Roger has $40 to buy CDs. The CDs cost $5 each. He will definitely buy at least 3 CDs. How many CDs can Roger buy? Use inequalities to solve the problems.

28. **Multi-Step** A summer school program has a budget of $1000 to buy T-shirts. Twenty free T-shirts will be received when they place their order. The number of T-shirts \(y\) that the program can get is given by \(y = \frac{1000}{x} + 20\), where \(x\) is the price per T-shirt.
   a. What is the horizontal asymptote of this rational function?
   b. What is the vertical asymptote?
   c. If the price per T-shirt is $10, how many T-shirts can the program receive?

*29. **Suppose the area of a rectangle is represented by the expression** \(4x^2 + 9x + 2\). Find possible expressions for the length and width of the rectangle.

30. Simplify the expression \(6\sqrt{8} \cdot \sqrt{5}\).
Warm Up

1. **Vocabulary** For any nonzero real number \( n \), the reciprocal of the number \( n \) is \( \frac{1}{n} \).

Identify the LCM.

2. \( 4x - 16 \) and \( x - 4 \)

3. \( 18x^2 \) and \( 9x \)

Factor.

4. \( x^2 - 4x - 77 \)

5. \( 18x^2 + 12x + 2 \)

New Concepts

A **complex fraction** is a fraction that contains one or more fractions in the numerator or the denominator.

Complex Fractions

There are two ways to write a fraction divided by a fraction.

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}, \quad \text{when} \quad b \neq 0, \ c \neq 0, \ \text{and} \ d \neq 0.
\]

A complex fraction can be written as a fraction divided by a fraction. The rules for dividing fractions can be applied to simplify complex fractions.

**Example 1** Simplifying by Dividing

Simplify \( \frac{\frac{a}{x}}{\frac{b}{a + x}} \).

**SOLUTION**

\[
\frac{\frac{a}{x}}{\frac{b}{a + x}} = \frac{a}{x} \div \frac{b}{a + x} \quad \text{Write using a division symbol.}
\]

\[
= \frac{a}{x} \cdot \frac{a + x}{b} \quad \text{Multiply by the reciprocal.}
\]

\[
= \frac{a(a + x)}{xb} \quad \text{Multiply.}
\]
The product of a number and its reciprocal is 1. To eliminate the fraction in the denominator of a complex fraction, multiply the numerator and the denominator by the reciprocal of the denominator.

**Example 2**  
**Simplifying Using the Reciprocal of the Denominator**

Simplify \( \frac{am}{n} \cdot \frac{x}{mn} \).

**SOLUTION**

\[
\frac{am}{n} \cdot \frac{x}{mn} = \frac{am \cdot mn}{n \cdot x \cdot mn} \cdot \frac{x}{mn} \quad \text{Multiply by the reciprocal of the denominator.}
\]

\[
= \frac{am^2}{x} \quad \text{Divide out common factors.}
\]

\[
= \frac{am^2}{x} \quad \text{Multiply.}
\]

\[
= \frac{am^2}{x} \quad \text{Simplify.}
\]

**Example 3**  
**Factoring to Simplify**

Simplify \( \frac{3x}{6x + 12} \cdot \frac{9}{x + 2} \).

**SOLUTION**

\[
\frac{3x}{6x + 12} \cdot \frac{9}{x + 2} = \frac{3x}{6(x + 2)} \cdot \frac{9}{x + 2} \quad \text{Write using a division symbol.}
\]

\[
= \frac{3x}{6(x + 2)} \cdot \frac{x + 2}{9} \quad \text{Factor out the GCF and multiply by the reciprocal.}
\]

\[
= \frac{1}{6} \cdot \frac{3x}{(x + 2)} \cdot \frac{x + 2}{9} \quad \text{Divide out common factors.}
\]

\[
= \frac{x}{18} \quad \text{Simplify.}
\]
Example 4  Combining Fractions to Simplify

Simplify \( \frac{1}{x} - \frac{1}{x} \).

**SOLUTION**

\[
\frac{1}{x} - \frac{1}{x} = \frac{1}{x} \cdot \frac{x - 1}{x} \quad \text{Subtract in the denominator.}
\]

\[
= \frac{1}{x} \div \frac{x - 1}{x} \quad \text{Write using a division symbol.}
\]

\[
= \frac{1}{x} \cdot \frac{x}{x - 1} \quad \text{Multiply by the reciprocal.}
\]

\[
= \frac{1}{x - 1} \quad \text{Divide out common factors.}
\]

\[
= \frac{1}{x - 1} \quad \text{Simplify.}
\]

Example 5  Application: Speed Walking

It took Max \( \frac{3x^2 - 12x}{3x} \) minutes to speed walk to the gym that was \( \frac{5x - 20}{x^3} \) miles away. Find his rate in miles per minute.

**SOLUTION**

\[
r = \frac{d}{t} \quad \text{Solve for } r.
\]

\[
r = \frac{\frac{5x - 20}{x^3}}{\frac{3x^2 - 12x}{3x}} \quad \text{Evaluate for } d \text{ and } t.
\]

\[
= \frac{3x}{x^3} \div \frac{3x^2 - 12x}{3x} \quad \text{Write using a division symbol.}
\]

\[
= \frac{5x - 20}{x^3} \cdot \frac{3x}{3x^2 - 12x} \quad \text{Multiply by the reciprocal.}
\]

\[
= \frac{5(x - 4)}{x^3} \cdot \frac{3x}{3x(x - 4)} \quad \text{Factor out any GCFs.}
\]

\[
= \frac{5}{x^3} \quad \text{miles per minute} \quad \text{Divide out common factors and simplify.}
\]

The expression \( \frac{5}{x^3} \) represents Max’s speed-walking rate in miles per minute.
Lesson Practice

Simplify.

a. \( \frac{x}{4} \cdot \frac{3(x - 3)}{x} \)  

b. \( \frac{b}{cd} \cdot \frac{2b}{c} \)

c. \( \frac{4x^2}{x - 3} \cdot \frac{x}{3x - 9} \)

d. \( \frac{1}{m} + 5 \cdot \frac{2}{m} - \frac{x}{m} \)

e. It took Ariel \( \frac{5x^2 - 45x}{5x} \) minutes to walk to school that was \( \frac{3x - 27}{x^3} \) miles away. Find her rate in miles per minute.

Practice Distributed and Integrated

Find the product or quotient.

1. \( \frac{15x^4}{x - 4} \cdot \frac{x^2 - 10x + 24}{3x^3 + 12x^2} \)  
2. \( \frac{x^2 + 12x + 36}{x^2 - 36} \div \frac{1}{x - 6} \)

Solve.

3. \( -3(r - 2) > -2(-6) \)  
4. \( \frac{y}{4} + \frac{1}{2} < \frac{2}{3} \)

Simplify.

5. \( \frac{5x}{10x + 20} \cdot \frac{15}{x + 2} \)
6. \( 8\sqrt{9} \cdot 2\sqrt{5} \)

*7. Write* Under what conditions is a rational expression undefined?

*8. Justify* Give an example to show \( \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \).

*9. (Skating)* It took Jim \( \frac{15}{x^2 + 2x - 3} \) minutes to skate to the park that was \( \frac{2x}{8x - 8} + \frac{x}{4x + 12} \) miles away. Find his rate in miles per minute.
10. **Multiple Choice** Two fractions have a denominator of \( x^2 + 6x + 9 \) and \( x^2 - 9 \). What is the least common denominator?
   - A \( x^2 + 9 \)
   - B \( x^2 + 6x + 9 \)
   - C \( 2x^2 + 9 \)
   - D \( (x + 3)^2(x - 3) \)

11. Solve and graph the inequality \( |x| < 65 \).

12. **Error Analysis** Two students solve the inequality \( |x - 15| < -4 \). Which student is correct? Explain the error.
   - Student A: \( |x - 15| < -4 \)
   - Student B: \( -4 < x - 15 < 4 \) no solution

13. **Geometry** A triangle has sides measuring 16 inches and 23 inches. The triangle inequality states that the length of the third side must be greater than 7 inches and less than 39 inches. Write this inequality and graph it.

14. **Multi-Step** The grades on a math test were all within the range of 80 points plus or minus 15 points.
   a. Write an absolute-value inequality to show the range of the grades.
   b. Solve the inequality to find the actual range of the grades.

15. **Error Analysis** Two students simplified \( \frac{x + 2}{2x - 5} - \frac{3 - x}{2x - 5} \). Which student is correct? Explain the error.
   - Student A: \( \frac{x + 2 - 3 + x}{2x - 5} = \frac{2x - 1}{2x - 5} \)
   - Student B: \( \frac{x + 2 - 3 - x}{2x - 5} = \frac{-1}{2x - 5} \)

16. **Canoeing** Vanya paddled a canoe upstream for 4 miles. He then turned the canoe around and paddled downstream for 3 miles. The current flowed at a rate of \( c \) miles per hour. Write a simplified expression to represent his total canoeing time if he kept a constant paddling rate of 6 miles per hour.

17. **Multiple Choice** Which equation’s graph has a maximum?
   - A \( y = -5 + x^2 \)
   - B \( y = -x^2 + 5x \)
   - C \( y = x^2 + 5 \)
   - D \( y = 5x^2 - 1 \)

18. **Write** Describe 2 ways to find the axis of symmetry for a parabola.

19. **Justify** Give an example of a quadratic function and an example of a function that is not quadratic. Explain why each function is or is not quadratic.
20. Find the length \(a\) to the nearest tenth.

21. Subtract \(\frac{5f + 6}{f^2 + 7f - 8} - \frac{f + 10}{f^2 + 7f - 8}\).

22. Let \(A = (-5, 3)\), \(B = (0, 7)\), \(C = (12, 7)\), and \(D = (7, 3)\). Use the distance formula to determine whether \(ABCD\) is a parallelogram.

23. Geography Ithaca, New York is almost directly west of Oneonta, New York and directly north of Athens, Pennsylvania. The three cities form a triangle that is nearly a right triangle. Use the distance formula to estimate the distance from Athens to Oneonta. Each unit on the grid represents 5 miles.

24. Error Analysis Two students factor the polynomial \((3x^2 + 6) - (4x^3 + 8x)\). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3(x^2 + 2) - 4x(x^2 - 2)}{(3 - 4x)(x^2 - 4)})</td>
<td>(\frac{3(x^2 + 2) - 4x(x^2 + 2)}{(3 - 4x)(x^2 + 2)})</td>
</tr>
</tbody>
</table>

25. Probability A bag has \(2x + 1\) red marbles, \(3x\) blue marbles, and \(x + 2\) green marbles. What is the probability of picking a red marble?

26. Multi-Step A toolbox is 2 feet high. Its volume is represented by the expression \(2x^2 - 8x + 6\).
   a. Factor the expression completely.
   b. Identify the expressions that represent the length and width of the toolbox.

27. Hardiness Zones The state of Kansas falls into USDA hardiness zones 5b and 6a. This means that plants in these zones must be able to tolerate an average minimum temperature range greater than or equal to \(-15^\circ\text{F}\), and less than \(-5^\circ\text{F}\). Write an inequality to represent the temperature range of the hardiness zones in Kansas.
28. **Multi-Step** A rectangular playing field has a perimeter of 8x feet. Its length is 2 feet greater than 2x.
   
   a. Write expressions for the length and width of the rectangle.
   
   b. Write an expression for the area of the field.
   
   c. The playing field is in a city park. The park is a large square with a side length of x. Write and factor an expression for the area of the park that does not include the playing field.

29. **Vertical Motion** The height \( h \) of an object \( t \) seconds after it begins to fall is given by the equation \( h = -16t^2 + vt + s \), where \( v \) is the initial velocity and \( s \) is the initial height. When an object falls, its initial velocity is zero. Write an equation for the height of an object \( t \) seconds after it begins to fall from 14,400 feet. Then factor the expression representing the height.

30. Given the equation \( r = \frac{kst}{p} \), use the terms “jointly proportional to” and “inversely proportional to” to describe the relation in the equation such that \( k \) is the constant of variation.
Dividing Polynomials

Warm Up

1. **Vocabulary** A __________ is a monomial or the sum or difference of monomials.

Divide.

2. \( \frac{72x^3 - 8x}{8x} \)

3. \( \frac{15x^2 - 3x}{5x - 1} \)

Factor.

4. \( 2x^2 + x - 3 \)

5. \( 25x^3 - 9 \)

New Concepts

A polynomial division problem can be written as a rational expression. To divide a polynomial by a monomial, divide each term in the numerator by the denominator.

**Example 1** Dividing a Polynomial by a Monomial

Divide \((8x^3 + 12x^2 + 4x) \div 4x\).

**SOLUTION**

\[
\frac{8x^3 + 12x^2 + 4x}{4x} = \frac{(8x^3 + 12x^2 + 4x)}{4x} \quad \text{Write as a rational expression.}
\]

\[
= \frac{8x^3}{4x} + \frac{12x^2}{4x} + \frac{4x}{4x} \quad \text{Divide each term by the denominator.}
\]

\[
= 2x^2 + 3x + 1 \quad \text{Simplify.}
\]

Division of a polynomial by a binomial is similar to division of whole numbers.

<table>
<thead>
<tr>
<th>Step</th>
<th>Words</th>
<th>Numbers</th>
<th>Polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Factor the numerator and denominator, if possible.</td>
<td>( \frac{128}{4} = \frac{32 \cdot 4}{4} )</td>
<td>( \frac{x^2 + 4x + 3}{x + 1} = \frac{(x + 3)(x + 1)}{x + 1} )</td>
</tr>
<tr>
<td>Step 2</td>
<td>Divide out any common factors.</td>
<td>( \frac{32 \cdot 4}{4} )</td>
<td>( \frac{(x + 3)(x + 1)}{x + 1} )</td>
</tr>
<tr>
<td>Step 3</td>
<td>Simplify.</td>
<td>32</td>
<td>( x + 3 )</td>
</tr>
</tbody>
</table>
**Example 2** Dividing a Polynomial by a Binomial

Divide each expression.

**a.** \((x^2 - 6x + 9) ÷ (x - 3)\)

**SOLUTION**

\[
\frac{x^2 - 6x + 9}{x - 3} = \frac{(x - 3)(x - 3)}{x - 3}
\]

Write as a rational expression.

\[
= \frac{(x - 3)(x - 3)}{x - 3}
\]

Factor the numerator.

\[
= \frac{(x - 3)}{(x - 3)}
\]

Divide out common factors.

\[
= x - 3
\]

Simplify.

**b.** \((x^2 - 5x + 6) ÷ (2 - x)\)

**SOLUTION**

\[
\frac{x^2 - 5x + 6}{2 - x} = \frac{(x - 3)(x - 2)}{2 - x}
\]

Write as a rational expression.

\[
= \frac{(x - 3)(x - 2)}{2 - x}
\]

Factor the numerator.

\[
= \frac{(x - 3)(x - 2)}{-1(x - 2)}
\]

Write the denominator in descending order.

\[
= \frac{(x - 3)}{-1(x - 2)}
\]

Factor out a \(-1\) in the denominator.

\[
= \frac{(x - 3)}{-1(x - 2)}
\]

Divide out common factors.

\[
= -x + 3
\]

Simplify.

**Hint**

When the signs of the binomials, one in the numerator and one in the denominator, are opposites, factor out \(-1\).

Just as with whole numbers, long division can also be used to divide polynomials.

\[
\begin{array}{c|cc}
  & 27 & x + 2 \\
\hline
21 & 567 & x + 1 \\
-42 & & x^2 + 3x + 2 \\
\hline
147 & & -x \\
-147 & & 2x + 2 \\
\hline
0 & & -(2x + 2)
\end{array}
\]
Example 3 Dividing a Polynomial Using Long Division

Divide using long division.

\((-25x + 3x^2 + 8) ÷ (x - 8)\)

**SOLUTION**

\[
\begin{align*}
(25x + 3x^2 + 8) ÷ (x - 8) &\quad \text{Write in long-division form with expressions in standard form.} \\
(x - 8)(3x^2 - 25x + 8) &\quad \text{Divide the first term of the dividend by the first term of the divisor to find the first term of the quotient.} \\
3x &\quad \text{Multiply the first term of the quotient by the binomial divisor. Write the product under the dividend. Align like terms.} \\
x - 8(3x^2 - 25x + 8) &\quad \text{Subtract the product from the dividend. Then bring down the next term in the dividend.} \\
-3x^2 + 24x &\quad \text{Repeat the steps to find each term of the quotient.} \\
-x + 8 &\quad \text{The remainder is 0.} \\
0 &
\end{align*}
\]

The quotient is \((3x - 1)\) remainder 0.

**Check** Multiply the quotient and the divisor.

\[
(3x - 1)(x - 8) = 3x^2 - 24x - x + 8 = 3x^2 - 25x + 8
\]

The divisor is not always a factor of the dividend. When it is not, the remainder will not be 0. The remainder can be written as a rational expression using the divisor as the denominator.

Example 4 Long Division with a Remainder

Divide using long division.

\((2x^2 - 9 - 7x) ÷ (-4 + x)\)
SOLUTION

\[(2x^2 - 9 - 7x) ÷ (-4 + x)\]

Write in long-division form with expressions in standard form.

\[x - 4 \overline{2x^2 - 7x - 9}\]

\[\frac{2x}{x - 4}\]

Divide the first term of the dividend by the first term of the divisor to find the first term of the quotient.

\[\frac{2x}{x - 4}\]

Multiply the first term of the quotient by the binomial divisor. Write the product under the dividend. Align like terms.

\[\frac{2x}{x - 4}\]

Subtract the product from the dividend. Then bring down the next term in the dividend.

\[\frac{2x + 1}{x - 4}\]

Repeat the steps to find each term of the quotient.

\[\frac{2x + 1}{x - 4}\]

The quotient is \(2x + 1 - \frac{5}{x - 4}\).

Example 5 Dividing a Polynomial with a Zero Coefficient

Divide \((-2x + 5 + 3x^3) ÷ (-3 + x)\).

SOLUTION

\[(-2x + 5 + 3x^3) ÷ (-3 + x)\]

Write each polynomial in standard form.

\[\overline{3x^3 - 2x + 5}\]

Write in long division form. Use 0\(x^2\) as a placeholder for the \(x^2\)-term.

\[\frac{3x^3}{x - 3}\]

\[\frac{9x^2}{x - 3}\]

\[\frac{25x + 5}{x - 3}\]

Multiply \(3x^2(x - 3)\). Then subtract. Bring down \(-2x\). \(9x^2 ÷ x = 9x\)

Multiply \(9x(x - 3)\). Then subtract. Bring down 5. \(25x ÷ x = 25\)

Multiply \(25(x - 3)\). Then subtract. The remainder is 80.

The quotient is \(3x^2 + 9x + 25 + \frac{80}{x - 3}\).
**Example 6** Application: Length of a Garden

Jim wants to find the length of the rectangular garden outside his office. The area is \((x^2 - 11x + 30)\) square feet. The width is \((x - 6)\) feet. What is the length of the garden?

**SOLUTION**

\[
l = \frac{A}{w}
\]

Solve for \(l\).

\[
l = \frac{x^2 - 11x + 30}{x - 6}
\]

Evaluate for \(A\) and \(w\).

\[
l = \frac{(x - 6)(x - 5)}{x - 6}
\]

Factor the numerator.

\[
l = \frac{(x - 6)(x - 5)}{x - 6}
\]

Divide out common factors.

\[
l = (x - 5)
\]

Simplify.

The length of the garden is \((x - 5)\) feet.

**Lesson Practice**

Divide each expression.

a. \((7x^4 + 7x^3 - 84x^2) ÷ 7x^2\) \((\text{Ex } 1)\)

b. \((x^2 - 10x + 25) ÷ (x - 5)\) \((\text{Ex } 2)\)

c. \((3x^2 - 14x - 5) ÷ (5 - x)\) \((\text{Ex } 2)\)

d. \((8x^2 + x^3 - 20x) ÷ (x - 2)\) \((\text{Ex } 3)\)

e. \((-3x^2 + 6x^3 + x - 33) ÷ (-2 + x)\) \((\text{Ex } 4)\)

f. \((6x + 5x^3 - 8) ÷ (x - 4)\) \((\text{Ex } 5)\)

g. Carlos wants to find the width of his rectangular deck. The area is \((x^2 - 10x + 24)\) square feet and the length is \((x - 4)\) feet. What is the width?

**Practice** Distributed and Integrated

1. Find the distance between \((-3, 2)\) and \((9, -3)\). Give the answer in simplest radical form.

2. Solve \(\frac{5}{16}v + \frac{3}{8} ≥ \frac{1}{2}\) and graph the solution.

Factor.

3. \(2x^2 + 12x + 16\) \((\text{Ex } 79)\)

4. \(3x^3 - 5x^2 - 9x + 15\) \((\text{Ex } 87)\)
5. Find the quotient: \( \frac{4x^3 + 42x^2 - 2x}{2x} \).

6. Find the axis of symmetry for the graph of the equation \( y = x^2 - 2x \).

Simplify.

\[
\begin{align*}
7. & \quad \frac{7x^4}{4x + 18} \\
8. & \quad \frac{1}{x^3} \div \frac{1}{x^3 + 1}
\end{align*}
\]

9. Write Explain how to check that \((5x + 6)\) is the correct quotient of \((15x^2 + 13x - 6) \div (3x - 1)\).

10. Justify Show that the quotient of \((x^2 - 4) \div (x + 2)\) can be found using two different methods.

11. Swimming The city has decided to open a new public pool. The area of the new rectangular pool is \((x^2 - 16x + 63)\) square feet and the width is \((x - 7)\) feet. What is the length?

12. Multiple Choice Simplify \( \frac{x^3 - 7x + 3x^2 - 21}{x + 3} \).

A \( x^2 - 7 \)

B \( x^3 - 7 \)

C \(-3\)

D \( \frac{x^2 - 7}{2x} \)

13. Error Analysis Students were asked to simplify \( \frac{6x^2 - 6x}{8x^2 + 8x} \div \frac{3x - 3}{4x^2 + 4x} \). Which student is correct? Explain the error.

**Student A**

\[
\begin{align*}
\frac{6x^2 - 6x}{8x^2 + 8x} & \cdot \frac{4x^2 + 4x}{3x - 3} \\
= & \frac{6x(x - 1)}{8x(x + 1)} \cdot \frac{4x(x + 1)}{3(x - 1)} \\
= & \frac{6x^2}{24x} \\
= & x
\end{align*}
\]

**Student B**

\[
\begin{align*}
\frac{6x^2 - 6x}{8x^2 + 8x} & \cdot \frac{4x^2 + 4x}{3x - 3} \\
= & \frac{6x(x - 1)}{8x(x + 1)} \cdot \frac{4x(x + 1)}{3(x - 1)} \\
= & \frac{24x^2}{24x} \\
= & x
\end{align*}
\]

14. Multi-Step Brent rode his scooter \( \frac{8x^2 - 48x}{24x^3} \) minutes to get to baseball practice that was \( \frac{7x - 42}{4x^2} \) miles away.

a. Find his rate in miles per minute.

b. If the rate is divided by \( \frac{1}{x} \), what is the new rate?

15. Geometry The area of a parallelogram is \( \frac{m + n}{5} \) square inches and the height is \( \frac{m^2 + n^2}{15} \) inches. What is the length of the base?

16. Solve and graph the inequality \(|x| > 84\).
17. (Census) The census of England and Wales has a margin of error of ±104,000 people. The 2001 census found the population to be 52,041,916. Write an absolute-value inequality to express the possible range of the population. Then solve the inequality to find the actual range for the population.

18. Error Analysis Two students solve the inequality \(|x + 11| > -15\). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>x + 11</td>
</tr>
<tr>
<td><em>x</em> can be any real number.</td>
<td><em>x</em> = (\emptyset)</td>
</tr>
</tbody>
</table>

19. Multiple Choice When simplifying \(\frac{1}{x^2 - 5x - 50} + \frac{1}{2x - 20}\), what is the numerator?

A 1  
B 2  
C \(x + 5\)  
D \(x + 7\)

20. Analyze Explain what can be done so that \(\frac{1}{3 - r}\) and \(\frac{5r}{r - 3}\) have like denominators.

21. Probability Mr. Brunetti writes quadratic equations on pieces of paper and puts them in a hat, and then tells his students each to choose two at random. After each student picks, his or her two papers go back in the hat. The functions are \(y = 4x^2 - 3x + 7\), \(y = -7 + x^2\), \(y = -2x + 6x^2\), \(y = 0.5x^2 + 1.1\), and \(y = -\frac{1}{2}x^2 + 7x + 5\). What is the probability of a student choosing two functions that have a minimum?

22. Error Analysis Two students divide the following rational expression. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{m^2}{6m^2} \div (m^2 + 2))</td>
<td>(\frac{m^2}{6m^2} \div (m^2 + 2))</td>
</tr>
<tr>
<td>(\frac{m^2}{6m^2} \cdot \frac{1}{m^2 + 2} = \frac{1}{6(m^2 + 2)})</td>
<td>(\frac{m^2}{6m^2} \cdot \frac{m^2 + 2}{1} = \frac{m^2 + 2}{6})</td>
</tr>
</tbody>
</table>

23. Cost A bakery sells specialty rolls by the dozen. The first dozen costs \(6b + b^2\) dollars. Each dozen thereafter costs \(4b + b^2\) dollars. If Marcello buys 4 dozen rolls, how much does he pay?

24. Find the vertical and horizontal asymptotes and graph \(y = \frac{4}{x + 2}\).

25. Flooring Theo is installing new kitchen tiles. The design on the tile includes a square within a square. The smaller square has a side length of \(s\) centimeters. The expression \(4s^2 + 12s + 9\) describes the area of the entire tile. What is the difference between the length of the tile and the length of the square within the tile?
26. **Multi-Step** Tony is sketching the view from the top of a 256-foot-tall observation tower and accidentally drops his pencil.

a. Use the formula \( h = -16t^2 + 256 \) to make a table of values showing the height \( h \) of the pencil, 1, 2, and 3 seconds after it is dropped.

b. Graph the function.

c. About how long does it take for the pencil to hit the ground?

27. **Pendulums** The time it takes a pendulum to swing back and forth depends on its length. The formula \( l = 2.45 \frac{r^2}{\pi} \) approximates this relationship. Graph the function using 3.14 for \( \pi \). Use the graph to estimate the time it takes a pendulum that is 1 meter long to swing back and forth.

28. **Write** Explain how to determine whether a triangle with side lengths 5, 7, and 10 is a right triangle.

29. **Multi-Step** A bag of marbles contains 3 red, 5 blue, 2 purple, and 4 clear marbles.

   a. Make a graph that represents the frequency distribution.

   b. What is the probability of drawing a red or a clear marble?

30. **Multi-Step** Given the following table, find the value of the constant of variation and complete the missing values in the table given that \( y \) varies directly with \( x \) and inversely with \( z \).

<table>
<thead>
<tr>
<th>( y )</th>
<th>( x )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>
Saxon Algebra 1
624
Warm Up

1. Vocabulary
   The _______ of a number $n$ is the distance from $n$ to 0 on a number line.

Simplify.

2. $| -9 | - 5$  
   $| 12 - 23 |$

Solve.

4. $6x - 7 = 11$  
   $5. 11x + 8 = 41$

New Concepts

To solve an absolute-value equation, begin by isolating the absolute value. Then use the definition of absolute value to write the absolute-value equation as two equations. Solve each equation, and write the solution set. There are often two answers to an absolute-value equation. The solutions can be graphed on a number line by placing a closed circle at each value in the solution set.

Example 1 Solving Equations with Two Operations

Solve each equation. Then graph the solution.

a. $\frac{|x|}{5} + 3 = 18$

SOLUTION

First isolate the absolute value. Write the equation so that the absolute value is on one side of the equation by itself.

$$\frac{|x|}{5} + 3 = 18$$

$$\frac{|x|}{5} = 15$$

Multiplication Property of Equality

$$5 \cdot \frac{|x|}{5} = 5 \cdot 15$$

$$|x| = 75$$

Simplify.

$x = 75$ or $x = -75$

Write as two equations without an absolute value.

The solution set is $\{-75, 75\}$.

Graph the solution on a number line.
b. \(4|x| - 9 = 15\)

**SOLUTION**

First isolate the absolute value.

\[
4|x| - 9 = 15
\]

\[
4|x| = 24 \quad \text{Add 9 to both sides.}
\]

\[
|x| = 6 \quad \text{Divide both sides by 4.}
\]

\(x = 6\) and \(x = -6\)  
Write as two equations without an absolute value.

The solution set is \(\{-6, 6\}\).

Graph the solution on a number line.

---

**Example 2  Solving Equations with More than Two Operations**

Solve each equation.

**a.** \[
\frac{5|x|}{2} + 4 = 4
\]

**SOLUTION**

\[
\frac{5|x|}{2} + 4 = 4
\]

\[
\frac{5|x|}{2} = 0 \quad \text{Subtract 4 from both sides.}
\]

\[
5|x| = 0 \quad \text{Multiply both sides by 2.}
\]

\[
|x| = 0 \quad \text{Divide both sides by 5.}
\]

Since the absolute value is equal to zero, there is only one solution. The solution set is \(\{0\}\).

**b.** \[
\frac{2|x|}{6} + 3 = 1
\]

**SOLUTION**

\[
\frac{2|x|}{6} + 3 = 1
\]

\[
\frac{2|x|}{6} = -2 \quad \text{Subtract 3 from both sides.}
\]

\[
2|x| = -12 \quad \text{Multiply both sides by 6.}
\]

\[
|x| = -6 \quad \text{Divide both sides by 2.}
\]

By the definition of absolute value, we know that there are no solutions to this equation. The absolute value is never negative. The solution set is empty. You can write this as \(\{\}\) or \(\emptyset\).
Example 3  Solving Equations with Operations Inside the Absolute-Value Symbols

Solve each equation.

a. \( |2x| + 9 = 15 \)

SOLUTION

\[ |2x| + 9 = 15 \]
\[ |2x| = 6 \] Subtract 9 from both sides.

Write the equation as two equations without the absolute value. Then solve both equations.

\[ 2x = 6 \quad \text{or} \quad 2x = -6 \]
\[ x = 3 \] Divide both sides by 2. \[ x = -3 \]

The solution set is \( \{3, -3\} \).

b. \( 6|x + 3| - 8 = 10 \)

SOLUTION

\[ 6|x + 3| - 8 = 10 \]
\[ 6|x + 3| = 18 \] Add 8 to both sides.
\[ |x + 3| = 3 \] Divide both sides by 6.

Write the equation as two equations without the absolute value and solve.

\[ x + 3 = 3 \quad \text{or} \quad x + 3 = -3 \]
\[ x = 0 \] Subtract 3 from both sides. \[ x = -6 \]

The solution set is \( \{0, -6\} \).

c. \( 5\left|\frac{x}{3} - 2\right| = 15 \)

SOLUTION

\[ 5\left|\frac{x}{3} - 2\right| = 15 \]
\[ \left|\frac{x}{3} - 2\right| = 3 \] Divide both sides by 5.

Write the equation as two equations without the absolute value and solve.

\[ \frac{x}{3} - 2 = 3 \quad \text{or} \quad \frac{x}{3} - 2 = -3 \]
\[ \frac{x}{3} = 5 \] Add 2 to both sides. \[ \frac{x}{3} = -1 \]
\[ x = 15 \] Multiply both sides by 3. \[ x = -3 \]

The solution set is \( \{15, -3\} \).
**Example 4  Application: Archery**

A 60-centimeter indoor archery target has several rings around a circular center. If the average diameter of a ring is \(d\), and an arrow landing in that ring scores \(p\) points, the inner and outer diameters of the ring are given by the absolute-value equation \(|d + 6p - 63| = 3\). Find the inner and outer diameters of the 8-point ring.

**SOLUTION**

\[
|d + 6p - 63| = 3
\]

\[
|d + 6 \cdot 8 - 63| = 3 \quad \text{Substitute 8 for } p.
\]

\[
|d + 48 - 63| = 3 \quad \text{Multiply.}
\]

\[
|d - 15| = 3 \quad \text{Subtract.}
\]

Write the absolute-value equation as two equations and solve.

\[
d - 15 = 3 \quad \text{or} \quad d - 15 = -3
\]

\[
d = 18 \quad \text{Add 15 to both sides.} \quad d = 12
\]

The inner diameter of the ring is 12 centimeters and the outer diameter of the ring is 18 centimeters.

**Lesson Practice**

Solve each equation. Then graph the solution.

**a.** \(\left| \frac{x}{7} \right| + 10 = 18\)

**b.** \(3| x | - 11 = 10\)

Solve each equation.

**c.** \(\frac{4|x|}{9} + 23 = 11\)

**d.** \(\frac{|x| + 3}{2} - 2 = 1\)

**e.** \(|7x| + 2 = 37\)

**f.** \(5| x + 1 | - 2 = 23\)

**g.** \(9 \left| \frac{x}{2} - 1 \right| = 45\)

**h. Investments** A factory produces items that cost $5 to make. The factory would like to invest $100 plus or minus $10 in the first batch. Use the equation \(|5x - 100| = 10\) to find the least and greatest number of items the factory can produce.
Add or subtract.

1. \( \frac{m^2}{m - 4} - \frac{16}{m - 4} \)  
2. \( \frac{-66}{w^2 - w - 30} + \frac{w}{w - 6} \)

*3. Write* Explain why some absolute-value equations have no solutions.

*4. Multiple Choice* The solution set \(-12, 60\) correctly solves which absolute-value equation?

A. \( 6 \left| \frac{x}{4} - 1 \right| = 42 \)  
B. \( -2 \left| \frac{x}{4} - 1 \right| = 16 \)

C. \( 8 \left| \frac{x}{3} - 2 \right| = 48 \)  
D. \( -5 \left| \frac{x}{6} - 4 \right| = -30 \)

5. (Refurbishing) Rudy has \( x + y \) junk cars in his lot. He fixes them up and sells each car for \( \frac{\$400 + \$100x}{y} \). If he sells 30% of them, how much profit will he make?

6. (Physics) The function \( y = -16x^2 + 80x \) models the height of a droplet of water from an in-ground sprinkler \( x \) seconds after it shoots straight up from ground level. Explain how you know when the droplet will hit the ground.

Factor.

7. \( 2a^2 + 8ab + 6a + 24b \)

8. \( 3x^{10} - 42x^9 - 21x^8 \)

9. Find the product of \( \frac{b - 4}{b + 9} \cdot (b^2 + 11b + 18) \).

*10. Error Analysis* Students were asked to simplify \( (15x^4 + 4x^2 + 3x^3) \div (x - 6) \). Which student set their problem up correctly? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (x - 6)(15x^4 + 4x^2 + 3x^3) )</td>
<td>( (x - 6)(15x^4 + 3x^3 + 4x^2 + 0x + 0) )</td>
</tr>
</tbody>
</table>

Simplify.

*11. \( \frac{-1}{10x - 10} \)

12. \( \frac{2x}{6x^2} \)

\( \frac{3x + 12}{x^2 + 8x + 16} \)
13. **Canoeing** A canoe rental company charges $10 for the canoe and an additional charge per person. There are 4 people going on the trip and they have planned on spending a total of $50. They hope that the total cost is within $20 of the planned spending total. What is the minimum and maximum they can be charged per person?

Find the quotient.
14. \(1 + 4x^4 - 10x^2\) \(\div\) \((x + 2)\)
15. \(\frac{25x^3 + 20x^2 - 5x}{5x}\)

16. Solve the equation \(\frac{|x|}{11} + 9 = 15\) and graph the solution.

17. **Justify** Show that the solution set to the equation \(\frac{|x|}{-3} + 1 = 5\) is \(\emptyset\).

18. **Multi-Step** Marty measured the area of his rectangular classroom. He determined that the area is \((-2x^2 + x^3 - 98 - 49x)\) square feet. The length is \((x + 6)\) feet.
   a. What is the width?
   b. If the area is \((x^2 - 36)\) square feet, what is the width?

19. **Geometry** The area of a triangle is \((10y^2 + 6y)\) square centimeters. The base is \((5y + 3)\) centimeters. What is the height?

20. **Error Analysis** Students were asked to simplify \(\frac{4x}{8x + 16}\) \(\div\) \(\frac{12}{x + 2}\). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\frac{4x}{8x + 16} \cdot \frac{12}{x + 2} = \frac{4x}{8(x + 2)} \cdot \frac{12}{x + 2} = \frac{x}{24}]</td>
<td>[\frac{4x}{8x + 16} \cdot \frac{x + 2}{12} = \frac{4x}{8(x + 2)} \cdot \frac{x + 2}{12} = \frac{x}{24}]</td>
</tr>
</tbody>
</table>

21. **Commuting** It took Taylor \(\frac{1}{x^2 + 3x - 40}\) minutes to get to work, which was \(\frac{1}{6x + 48}\) miles away. Find his rate in miles per minute.

22. **Verify** Show that 0 is a solution to the inequality \(|x - 14| < 30\).

23. **Multiple Choice** Which inequality is represented by the graph?

A | \(|x| \leq -21\)  
B | \(|x| < 21\)  
C | \(|x| \leq 21\)  
D | \(|x| > 21\)

24. **Analyze** When a line segment is horizontal, which expression under the radical in the distance formula is 0: \((x_2 - x_1)^2\) or \((y_2 - y_1)^2\)?
25. **Multi-Step** Ornella hiked 8 miles on easy trails and 3 miles on difficult trails. Her hiking rate on the easy trails was 2.5 times faster than her rate on the difficult trails.
   a. Write a simplified rational expression for Ornella’s total hiking time.
   b. Find Ornella’s hiking time if her hiking rate on the difficult trails was 2 miles per hour.

26. Find the vertical and horizontal asymptotes and graph \( y = \frac{1}{x - 2} - 4 \).

27. **Multi-Step** Phone Plan A costs $12 per month for local calls and $0.06 per minute for long-distance calls. Phone Plan B costs $15 per month for local calls plus $0.04 per minute for long-distance calls. How many minutes of long-distance calls would Jenna have to make for Plan B to cost less than Plan A?
   a. **Formulate** Write an inequality to answer.
   b. Solve the inequality and answer the question.
   c. Graph the solution.

28. **(United States Flag)** An official American flag should have a length that is 1.9 times its width. The area of an official American flag can be found by the function \( y = 1.9x^2 \). Graph the function. Then use the graph to approximate the width of a flag that has an area of 47.5 square feet.

29. **Multi-Step** A rectangular garden that is 25 feet wide has a diagonal length that is 50 feet long.
   a. Find the length of the garden in simplest radical form.
   b. Find the perimeter of the garden to the nearest tenth of a foot.

30. The volume of a sphere is \( V = \frac{4}{3} \pi r^3 \). Describe in words the relationship between the volume of a sphere and its radius. Identify the constant of variation.
Combining Rational Expressions with Unlike Denominators

Warm Up

1. **Vocabulary** One of two or more numbers or expressions that are multiplied to get a product is called a (n) ________.

   Find the LCM.

2. \(8x^4y\) and \(12x^3y^2\)  
3. \((9x - 27)\) and \((4x - 12)\)

   Factor.

4. \(x^2 + 4x - 21\)
5. \(10x^2 + 13x - 3\)

New Concepts

The steps for adding rational expressions are the same as for adding numerical fractions. Fractions with unlike denominators cannot be added unless you first find their least common denominator.

**Example 1** Finding a Common Denominator

Find the least common denominator (LCD) for each expression.

\[
\begin{align*}
\text{a. } & \quad \frac{3}{x + 3} \quad \frac{9}{x^2 - 2x - 15} \\
\text{SOLUTION} & \quad \frac{3}{x + 3} - \frac{9}{x^2 - 2x - 15} \\
& \quad = \frac{3}{x + 3} - \frac{9}{(x + 3)(x - 5)} \quad \text{Factor each denominator, if possible.} \\
& \quad \text{To find the LCD of } (x + 3) \text{ and } (x + 3)(x - 5), \text{ use every factor of} \\
& \quad \text{each denominator the greatest number of times it is a factor of either} \\
& \quad \text{denominator. Each denominator has a factor of } (x + 3). \text{ One denominator} \\
& \quad \text{also has a factor of } (x - 5). \text{ The product of these two factors is the LCD.} \\
& \quad \text{LCD} = (x + 3)(x - 5) \\
\end{align*}
\]

\[
\begin{align*}
\text{b. } & \quad \frac{2x}{4x^2 - 196} + \frac{12x}{x^2 + x - 56} \\
\text{SOLUTION} & \quad \frac{2x}{4x^2 - 196} + \frac{12x}{x^2 + x - 56} \\
& \quad = \frac{2x}{4(x - 7)(x + 7)} + \frac{12x}{(x - 7)(x + 8)} \quad \text{Factor each denominator} \\
& \quad \text{completely.} \\
& \quad \text{LCD} = 4(x - 7)(x + 7)(x + 8)
\end{align*}
\]
**Example 2** Using Equivalent Fractions to Add with Unlike Denominators

Add \( \frac{6x^2}{x^2 - 16} + \frac{x - 1}{2x - 8} \).

**SOLUTION** Factor each denominator.

\[
\frac{6x^2}{x^2 - 16} + \frac{x - 1}{2x - 8} = \frac{6x^2}{(x - 4)(x + 4)} + \frac{x - 1}{2(x - 4)}
\]

LCD = \(2(x - 4)(x + 4)\)

Write an equivalent fraction for each addend with the LCD as a denominator.

\[
\frac{6x^2}{(x - 4)(x + 4)} \cdot \frac{2}{2} = \frac{2(6x^2)}{2(x - 4)(x + 4)}
\]

Multiply the numerator and denominator of the first fraction by 2.

\[
\frac{x - 1}{2(x - 4)} \cdot \frac{x + 4}{x + 4} = \frac{(x - 1)(x + 4)}{2(x - 4)(x + 4)}
\]

Multiply the numerator and denominator of the second fraction by \(\frac{x + 4}{x + 4}\).

Write the sum of the equivalent fractions.

\[
\frac{2(6x^2)}{2(x - 4)(x + 4)} + \frac{(x - 1)(x + 4)}{2(x - 4)(x + 4)}
\]

Add.

\[
= \frac{2(6x^2) + (x - 1)(x + 4)}{2(x - 4)(x + 4)}
\]

Expand the numerator.

\[
= \frac{12x^2 + x^3 + 3x - 4}{2(x - 4)(x + 4)}
\]

Combine like terms in the numerator.

\[
= \frac{13x^2 + 3x - 4}{2(x - 4)(x + 4)}
\]

**Example 3** Using Equivalent Fractions to Subtract with Unlike Denominators

Subtract \( \frac{4x^2}{9x - 27} - \frac{2x - 5}{x^2 - 9} \).

**SOLUTION** Factor each denominator.

\[
\frac{4x^2}{9x - 27} - \frac{2x - 5}{x^2 - 9} = \frac{4x^2}{9(x - 3)} - \frac{2x - 5}{(x - 3)(x + 3)}
\]

LCD = \(9(x - 3)(x + 3)\)

\[
\frac{4x^2(x + 3)}{9(x - 3)(x + 3)} - \frac{9(2x - 5)}{9(x - 3)(x + 3)} \quad \text{Write the difference of the equivalent fractions.}
\]

\[
= \frac{4x^2(x + 3) - 9(2x - 5)}{9(x - 3)(x + 3)} \quad \text{Subtract.}
\]

\[
= \frac{4x^2 + 12x^2 - 18x + 45}{9(x - 3)(x + 3)} \quad \text{Expand the numerator.}
\]

There are no like terms, so the difference is \(\frac{4x^2 + 12x^2 - 18x + 45}{9(x - 3)(x + 3)}\).
Example 4 Adding and Subtracting with Unlike Denominators

(a) Add \( \frac{-35}{56 - 7x} + \frac{-6x - 2}{x^2 - 6x - 16} \).

**SOLUTION**

\[
\frac{-35}{56 - 7x} + \frac{-6x - 2}{x^2 - 6x - 16} = \frac{-35}{-7(x - 8)} + \frac{-6x - 2}{(x - 8)(x + 2)}
\]

Factor each denominator.

\[
\text{LCD} = -7(x - 8)(x + 2)
\]

Find the LCD.

\[
\frac{-35(x + 2)}{-7(x - 8)(x + 2)} + \frac{-7(-6x - 2)}{-7(x - 8)(x + 2)}
\]

Write each fraction as an equivalent fraction.

\[
= \frac{-35(x + 2) + (-7)(-6x - 2)}{-7(x - 8)(x + 2)}
\]

Add.

\[
= \frac{7x - 56}{-7(x - 8)(x + 2)}
\]

Expand the numerator and collect like terms.

\[
= \frac{7(x - 8)}{-7(x - 8)(x + 2)}
\]

Factor the numerator.

\[
= \frac{1}{x + 2}
\]

Divide out common factors.

(b) Subtract \( \frac{x - 5}{2x - 6} - \frac{x - 7}{4x - 12} \).

**SOLUTION**

\[
\frac{x - 5}{2x - 6} - \frac{x - 7}{4x - 12} = \frac{x - 5}{2(x - 3)} - \frac{x - 7}{4(x - 3)}
\]

Factor each denominator.

\[
\text{LCD} = 4(x - 3)
\]

Find the LCD.

\[
\frac{2(x - 5)}{4(x - 3)} - \frac{(x - 7)}{4(x - 3)}
\]

Write each fraction as an equivalent fraction.

\[
= \frac{2(x - 5) - 1(x - 7)}{4(x - 3)}
\]

Subtract.

\[
= \frac{2x - 10 - x + 7}{4(x - 3)}
\]

Expand the numerator.

\[
= \frac{x - 3}{4(x - 3)}
\]

Collect like terms.

\[
= \frac{1}{4}
\]

Divide out common factors.
Example 5  Application: Traveling

A pilot’s single-engine aircraft flies at 230 kph if there is no wind. The pilot plans a round trip to a city that is 400 kilometers away. If there is a tailwind of \( w \) kilometers per hour, the time for the outbound flight is \( \frac{400}{230 + w} \) hours. The time for the return flight with a headwind of \( w \) kilometers per hour is \( \frac{400}{230 - w} \) hours. What is the total time for the round trip?

**SOLUTION**

\[
\frac{400}{230 + w} + \frac{400}{230 - w}
\]

Add to find the total time.

**LCD**

\[
\text{Add to find the total time.}
\]

\[
\frac{400(230 - w)}{(230 + w)(230 - w)} + \frac{400(230 + w)}{(230 + w)(230 - w)}
\]

Find the LCD.

\[
= \frac{184,000}{(230 + w)(230 - w)}
\]

Write equivalent fractions.

\[
= \frac{184,000}{(230 + w)(230 - w)} \text{ hours}
\]

Expand the numerator and collect like terms.

The round trip takes \( \frac{184,000}{(230 + w)(230 - w)} \) hours.

**Lesson Practice**

Find the LCD for each expression.

**a.** \( \frac{5x}{5x - 45} - \frac{44}{x^2 - 81} \)

**b.** \( \frac{3x}{x + 4} - \frac{12}{x^2 + 2x - 8} \)

**c.** Add \( \frac{3x^2 - 25}{x^2 - 81} + \frac{x - 1}{4x - 20} \)

**d.** Subtract \( \frac{2x^2}{6x - 24} - \frac{3x - 4}{x^2 - 16} \)

**e.** Add \( \frac{x - 1}{x^2 - 1} + \frac{2}{5x + 5} \)

**f.** Subtract \( \frac{2}{x^2 - 36} - \frac{1}{x^2 + 6x} \)

**g.** Trenton hiked \( \frac{4x}{x^2 - 64} \) miles on Saturday and \( \frac{12}{7x - 56} \) miles on Sunday. How many miles did he hike altogether?

**Practice**

Factor completely.

**1.** \( 3x^3 - 9x^2 - 30x \)

**2.** \( 8x^4y^2 + 4x^3y - 12xy^3 \)

**3.** \( 32x^3 - 24x^4 + 4x^5 \)

**4.** \( mn^3 - 10mn^2 + 24mn \)
Find the quotient.

5. \( \frac{4m}{17r} \div \frac{12m^2}{5r} \)  

6. \( (x^2 - 16x + 64) \div (x - 8) \)

7. Find the zeros of the function shown.

*8. Find the LCD of \( \frac{4}{x + 4} - \frac{8}{x^2 + 6x + 8} \).

*9. Add \( \frac{2x}{2x^2 - 128} + \frac{5}{x^2 - 7x - 8} \).

*10. Solve the equation \(|x| - 22 = 14\) and graph the solution.

11. Error Analysis Students were asked to subtract \( \frac{6x^2}{x^2 + 6x - 16} - \frac{3}{8x - 16} \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{6x^2}{x^2 + 6x - 16} )</td>
<td>( \frac{6x^2}{x^2 + 6x - 16} )</td>
</tr>
<tr>
<td>( \frac{8x - 16}{x^2 + 6x - 16} )</td>
<td>( \frac{8x - 16}{x^2 + 6x - 16} )</td>
</tr>
<tr>
<td>( \frac{(x + 8)(x - 2)}{6x^2} )</td>
<td>( \frac{8(x - 2)}{6x^2} )</td>
</tr>
<tr>
<td>( \frac{6x^2(8)}{8(x - 2)} )</td>
<td>( \frac{3(x + 8)}{8(x - 2)} )</td>
</tr>
<tr>
<td>( \frac{8(x + 8)(x - 2)}{8(x - 2)(x + 8)} )</td>
<td>( \frac{8(x + 8)(x - 2)}{8(x - 2)(x + 8)} )</td>
</tr>
<tr>
<td>( \frac{48x^2 - 3x - 24}{8(x + 8)(x - 2)} )</td>
<td>( \frac{48x^2 - 3x + 24}{8(x + 8)(x - 2)} )</td>
</tr>
</tbody>
</table>

12. Generalize Explain how to find the LCD of two algebraic rational expressions.

13. Running Michele is training for a marathon. She ran \( \frac{3x^2}{x^2 - 100} \) miles on Monday and \( \frac{x - 1}{2x - 20} \) miles on Tuesday. How many miles did she run in all?

14. Multi-Step The girls’ track team sprinted \( \frac{2x}{4x^2 - 196} \) meters Thursday and \( \frac{12x}{x^2 + x - 56} \) meters Friday.

a. What was the total distance that the track team sprinted?

b. If their rate was \( \frac{2x}{x + 8} \) meters per minute, how much time did it take them to sprint on Thursday and Friday?
**15. Error Analysis** Two students solve the equation $6|x + 3| - 8 = -2$. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6</td>
<td>x + 3</td>
</tr>
<tr>
<td>no solution</td>
<td>$6</td>
</tr>
<tr>
<td>Absolute values cannot equal a negative number.</td>
<td>$</td>
</tr>
<tr>
<td>$x + 3 = 1$ and $x + 3 = -1$</td>
<td>$x = -2$ and $x = -4$</td>
</tr>
</tbody>
</table>

**16. Geometry** The perimeter of a square must be 34 inches plus or minus 2 inches. What is the longest and the shortest length each side can be?

**17. Multi-Step** A student budgets $35 for lunch and rides each week. He gives his friend $5 for gas and then pays for 5 lunches a week. He has a $2 cushion in his budget, meaning that he can spend $2 more or less than he budgeted.

a. Write an absolute-value equation for the minimum and maximum he can spend on each lunch.

b. What is the maximum and the minimum he can spend on each lunch?

**18. Error Analysis** Students were asked to simplify $\frac{x^2 + 10x + 24}{x}$. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{(x + 6)(x + 4)}{x} = (x + 6)(4)$</td>
<td>$\frac{(x + 6)(x + 4)}{x}$</td>
</tr>
</tbody>
</table>

**19. Carpeing** The rectangular public library is getting new carpet. The area of the room is $(x^3 - 18x^2 + 81x)$ square feet. The length of the room is $(x - 9)$ feet. What is the width?

**20. Justify** Give an example to show $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

**21. Multiple Choice** Simplify: $\frac{x^2 - 9}{x^2 - 5x + 6} \cdot \frac{x^2 + 5x + 6}{x^2 - 4}$.

A 1  B $\frac{(x - 3)^2}{(x - 2)^2}$  C $-1$  D $\frac{x - 3}{x - 2}$

**22. Solve and graph the inequality** $|x| > 17$.

**23. Measurement** When measuring something in centimeters, the accuracy is within 0.1 centimeter. A board is measured to be 15.6 centimeters. The accuracy of the measurement can be represented by the absolute-value inequality $|x - 15.6| \leq 0.1$. Solve the inequality.
24. **Multi-Step** Reggie has made a stew that is at a temperature of 100°F, and he either wants to heat it up to 165°F or cool it down to 45°F. To heat up the stew will take at most 45 minutes plus 1 minute for each degree the temperature is raised. To cool down the stew will take at least 10 minutes plus 2 minutes for each degree the temperature is lowered. How much time does Reggie need to allow for changing the food temperature either up or down?
   a. How many degrees up and down does the stew need to go?
   b. How much time will it take to cool down or heat up the stew?
   c. Is it faster to heat up or cool down the stew?

25. **Olympic Swimming Pool** An Olympic swimming pool must have a width of 25 meters and a length of 50 meters. Find the length of a diagonal of an Olympic swimming pool to the nearest tenth of a meter.

26. **Multi-Step** A graphing calculator screen is 128 pixels wide and 240 pixels long. The “pixel coordinates” of three points are shown.

   ![Diagram of points](image)

   a. Find the distance in pixels between each pair of points.
   b. List the line segments in order from shortest to longest.

27. **Write** Create a polynomial in which each term has a common factor of 4a, and then factor the expression.

28. **Hiking** In celebration of getting to the top of Beacon Rock in southern Washington, Renata throws her hat up and off the top of Beacon Rock. The height of the hat x seconds after the throw (in meters) can be approximated by the function \( y = -5x^2 + 10x + 260 \). After how many seconds will the hat be at its maximum height? What is this height?

29. **Commuting** Mr. Shakour’s round trip commute to and from work totals 30 miles. Because of traffic, his speed on the way home is 5 miles per hour less than what it is on the way to work. Write a simplified expression to represent Mr. Shakour’s total commuting time.

30. A teacher randomly picks a shirt and a skirt from her closet to wear to school. Use the table to find the theoretical probability of the teacher choosing an outfit with a blue shirt and khaki skirt.

<table>
<thead>
<tr>
<th>Shirts</th>
<th>Red</th>
<th>Blue</th>
<th>Blue</th>
<th>White</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>Khaki</td>
<td>KR</td>
<td>KB</td>
<td>KB</td>
<td>KW</td>
<td>KW</td>
</tr>
<tr>
<td>Navy</td>
<td>NR</td>
<td>NB</td>
<td>NB</td>
<td>NW</td>
<td>NW</td>
</tr>
<tr>
<td>Black</td>
<td>BR</td>
<td>BB</td>
<td>BB</td>
<td>BW</td>
<td>BW</td>
</tr>
<tr>
<td>White</td>
<td>WR</td>
<td>WB</td>
<td>WB</td>
<td>WW</td>
<td>WW</td>
</tr>
</tbody>
</table>
Warm Up

1. **Vocabulary** A(n) __________ is a line that divides a figure or graph into two mirror-image halves.

Evaluate for the given value.

2. \(y = 4x^2 - 6x - 4\) for \(x = \frac{1}{2}\)

3. \(y = -x^2 + 5x - 6\) for \(x = -2\)

Find the axis of symmetry using the formula.

4. \(y = -2x^2 + 4x - 5\)

5. \(y = x^2 - 3x - 4\)

New Concepts

A quadratic function can be graphed using the axis of symmetry, the vertex, the \(y\)-intercept, and pairs of points that are symmetric about the axis of symmetry. The quadratic function in the standard form \(f(x) = ax^2 + bx + c\) can be used to find these parts of the graph of the parabola.

The equation of the axis of symmetry and the \(x\)-coordinate of the vertex of a quadratic function is \(x = -\frac{b}{2a}\). To find the \(y\)-coordinate of the vertex, substitute the \(x\)-coordinate of the vertex into the function. The \(y\)-intercept of a function is the point on the graph where \(x = 0\). For quadratic functions in standard form, the \(y\)-intercept is \(c\).

**Example 1** Graphing Quadratics of the Form \(y = x^2 + bx + c\)

Graph the function.

\(y = x^2 + 4x + 5\)

**SOLUTION**

**Step 1:** Find the axis of symmetry.

\[x = -\frac{b}{2a}\]

\[= -\frac{4}{2 \cdot 1} = -\frac{4}{2} = -2\]

Substitute values for \(b\) and \(a\).

The axis of symmetry is \(x = -2\).

**Step 2:** Find the vertex.

\(y = x^2 + 4x + 5\)

\[= (-2)^2 + 4(-2) + 5 = 1\] Substitute \(-2\) for \(x\).

The vertex is \((-2, 1)\).

**Step 3:** Find the \(y\)-intercept.

The \(y\)-intercept is \(c\), or 5.
**Step 4:** Find one point not on the axis of symmetry.

\[ y = x^2 + 4x + 5 \]

\[ y = (1)^2 + 4(1) + 5 = 10 \quad \text{Substitute 1 for } x. \]

A point on the curve is \((1, 10)\).

**Step 5:** Graph.

Graph the axis of symmetry \(x = -2\), the vertex \((-2, 1)\), the \(y\)-intercept \((0, 5)\). Reflect the point \((1, 10)\) over the axis of symmetry and graph the point \((-5, 10)\). Connect the points with a smooth curve.

**Example 2** Graphing Quadratics of the Form \(y = ax^2 + bx + c\)

Graph the function.

\[ y = 3x^2 + 18x + 13 \]

**SOLUTION**

**Step 1:** Find the axis of symmetry.

\[ x = -\frac{b}{2a} \quad \text{Use the formula.} \]

\[ = -\frac{18}{2(3)} = -3 \quad \text{Substitute values for } b \text{ and } a. \]

The axis of symmetry is \(x = -3\).

**Step 2:** Find the vertex.

\[ y = 3x^2 + 18x + 13 \]

\[ = 3(-3)^2 + 18(-3) + 13 = -14 \quad \text{Substitute } -3 \text{ for } x. \]

The vertex is \((-3, -14)\).

**Step 3:** Find the \(y\)-intercept.

The \(y\)-intercept is \(c\), or 13.

**Step 4:** Find one point not on the axis of symmetry.

\[ y = 3x^2 + 18x + 13 \]

\[ = 3(-1)^2 + 18(-1) + 13 = -2 \quad \text{Substitute } -1 \text{ for } x. \]

A point on the curve is \((-1, -2)\).

**Step 5:** Graph.

Graph the axis of symmetry \(x = -3\), the vertex \((-3, -14)\), the \(y\)-intercept \((0, 13)\). Reflect the point \((-1, -2)\) across the axis of symmetry to get the point \((-5, -2)\). Connect the points with a smooth curve.
### Example 3  Graphing Quadratics of the Form $y = ax^2 + c$

Graph the function.

$y = 5x^2 + 4$

**SOLUTION**

**Step 1:** Find the axis of symmetry.

$x = \frac{-b}{2a} = \frac{-0}{2(5)} = 0$

The axis of symmetry is $x = 0$.

**Step 2:** Find the vertex.

$y = 5x^2 + 4$

$= 5(0)^2 + 4 = 4$

Substitute 0 for $x$.

The vertex is $(0, 4)$.

**Step 3:** Find the $y$-intercept.

The $y$-intercept is $c$, or 4.

**Step 4:** Find one point not on the axis of symmetry.

$y = 5x^2 + 4$.

$= 5(-1)^2 + 4 = 9$

Substitute $-1$ for $x$.

A point on the curve is $(-1, 9)$

**Step 5:** Graph.

Graph the axis of symmetry $x = 0$, the vertex $(0, 4)$, the $y$-intercept $(0, 4)$. Reflect the point $(-1, 9)$ across the axis of symmetry to get the point $(1, 9)$. Connect the points with a smooth curve.

A zero of a function is an $x$-value for a function where $f(x) = 0$. It is the point where the graph of the function meets or intersects the $x$-axis. The **standard form of a quadratic equation** $ax^2 + bx + c = 0$, where $a \neq 0$, is the related equation to the quadratic function. The quadratic equation is used to find the zeros of a quadratic function algebraically. Alternatively, a graphing calculator can help find zeros of a quadratic function.

### Example 4  Finding the Zeros of a Quadratic Function

Find the zeros of the function.

a. $y = x^2 - 6x + 9$

**SOLUTION**

Use a graphing calculator to graph $y = x^2 - 6x + 9$.

The zero of the function is 3.
Lesson 96

If a quadratic function has no real zeros, then there are no real numbers that when substituted for \( x \) result in \( y = 0 \). The graph of such a function does not cross the \( x \)-axis.

**Math Language**

\[ b. \quad y = x^2 - 3x - 10 \]

**SOLUTION**

Use a graphing calculator to graph \( y = x^2 - 3x - 10 \).

There are two zeros for this function, 5 and -2.

\[ c. \quad y = -2x^2 - 3 \]

**SOLUTION**

Use a graphing calculator to graph \( y = -2x^2 - 3 \).

There are no real zeros for this function.

When an object is thrown or kicked into the air, it follows a parabolic path.

You can calculate its height in feet after \( t \) seconds using the formula \( h = -16t^2 + vt + s \). The initial vertical velocity in feet per second is \( v \), and \( s \) is the starting height of the object in feet.

**Example 5 Application: Baseball**

A baseball is thrown straight up with an initial velocity of 50 feet per second. The ball leaves the player’s hand when it is 4 feet above the ground. At what time does the ball reach its maximum height?

**SOLUTION**

Substitute the values given for the initial velocity and starting height into the formula \( h = -16t^2 + vt + s \). Then find the \( x \)-coordinate of the vertex. Use the formula \( x = \frac{-b}{2a} \).

\[
\begin{align*}
h &= -16t^2 + 50t + 4 \\
x &= \frac{-b}{2a} \\
&= \frac{-50}{2(-16)} = 1.5625 \quad \text{Substitute values for } b \text{ and } a.
\end{align*}
\]

The ball reaches its maximum height 1.5625 seconds after it has been thrown.
Lesson Practice

Graph each function.

- **a.** \( y = x^2 - 4x + 7 \)  
  - **b.** \( y = 2x^2 - 16x + 24 \)  
  - **c.** \( y = 2x^2 - 9 \)  

Find the zeros of each function.

- **d.** \( y = x^2 + 10x + 25 \)  
  - **e.** \( y = 3x^2 - 21x + 30 \)  
  - **f.** \( y = -\frac{1}{2}x^2 - 1 \)  
  - **g.** Soccer The height of a soccer ball that is kicked can be modeled by the function \( f(x) = -8x^2 + 24x \), where \( x \) is the time in seconds after it is kicked. Find the time it takes the ball to reach its maximum height.

Practice  Distributed and Integrated

1. Find the zeros of the function shown.

2. Add \( \frac{25}{16x^2y} + \frac{xy}{32y^3} \).

3. Solve the equation \( \frac{10|x|}{3} + 18 = 4 \) and graph the solution.

4. Solve \(-0.3 + 0.14n = 2.78\).

5. Solve \( \frac{6}{x-3} = \frac{3}{10} \).

6. Find the LCD of \( \frac{6}{x+6} - \frac{12}{x^2 + 8x + 12} \).

7. The table lists the ordered pairs from a relation. Determine whether they form a function.

<table>
<thead>
<tr>
<th>Domain (x)</th>
<th>Range (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

8. Simplify \( \frac{-x^5}{21x + 3} \).

9. Graph the function \( y = x^2 - 2x - 8 \).

10. Write Explain how to reflect a point across the axis of symmetry to get a second point on the parabola.
11. **Justify** Show that the vertex of \( y = 4x^2 - 24x + 9 \) is (3, -27).

12. **Multiple Choice** Which function has the vertex (6, -160)?
   A \( y = 6x^2 - 72x + 56 \)  
   B \( y = 2x^2 - 8x + 48 \)  
   C \( y = 3x^2 + 42x - 12 \)  
   D \( y = 5x^2 - 5x + 43 \)

13. **Diving** A diver moves upward with an initial velocity of 10 feet per second.
   How high will he be 0.5 seconds after diving from a 6-foot platform? Use \( h = -16t^2 + vt + s \).

14. **Multiple Choice** Find the LCD of \( \frac{3}{2x - 10} \) and \( \frac{5x}{2x^2 - 4x - 30} \).
   A \( 2(x - 5)(x + 3) \)  
   B \( (x - 5)(x + 3) \)  
   C \( (x + 5)(x - 3) \)  
   D \( \frac{2}{(x - 5)(x + 3)} \)

15. **Geometry** One side of a triangle is \( \frac{2}{x + 2} \) yards and two sides are each \( \frac{5}{3x + 6} \) yards. Find the perimeter of the triangle.

16. **Measurement** Carrie measured a distance of \( \frac{3x^2 - 9x - 18}{x^2 - 4} \) yards and Jessie measured a distance of \( \frac{4x - 5}{x^2 - 4} \) yards. How much longer is Carrie’s measurement than Jessie’s?

17. **Banking** The dollar amount in a student’s banking account is represented by the absolute-value inequality \( |x - 200| \leq 110 \). Solve the inequality and graph the solution.

18. **Generalize** Why is a place holder needed for missing variables in a polynomial dividend?

19. **Multiple Choice** Simplify \( (-5x + 2x^2 - 3) \div (x - 3) \).
   A \( 2x - 1 \)  
   B \( 2x + 1 \)  
   C \( \frac{x - 2}{2x} \)  
   D \( \frac{x^2 - 3}{5x} \)

20. **Physics** A family is going to see friends that live in two different towns. They will have to travel 100 miles plus or minus 10 miles to see either of them. They want to spend 2 hours in the car. What are the minimum and maximum rates that they need to go?

21. **Error Analysis** Two students graph the solution to the equation \( |2x + 10| = 8 \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>{-9, -1}</td>
<td>{-9, -1}</td>
</tr>
</tbody>
</table>

22. **Art** Jeremy’s picture frame has an area of \( x^2 - 18x + 80 \) square inches. He has two square pictures in it, one measuring \( \frac{1}{4} \) inch on each side and the other measuring \( \frac{1}{2} \) inch on each side. Write a simplified expression for the total fraction of the frame covered by pictures.
23. **Verify** Divide the rational expression \( \frac{3x^2 + 2x}{9y} \div \frac{y + 2}{y} \). How can you check your answer?

24. **Multi-Step** The volume of a prism is \( 6x^3 + 14x^2 + 4x \) cm\(^3\). Find the dimensions of the prism.
   a. Factor out common terms.
   b. Factor completely to find the dimensions.

25. **Baseball** A baseball diamond is a square that is 90 feet long on each side. To use a coordinate grid to model positions of players on the field, place home plate at \((0, 0)\), first base at \((90, 0)\), second base at \((90, 90)\), and third base at \((0, 90)\). An outfielder located at \((150, 80)\) throws to the first baseman. How long is the throw? Round your answer to the nearest foot.

26. **Multi-Step** A square has a side length of \( a \) centimeters. A smaller square has a side length of \( b \) centimeters.
   a. If the difference in the areas of the squares is \( a^2 - 16 \), what is the value of \( b \)?
   b. If the area of the larger square is \( 36b^2 + 60b + 25 \), what is the side length in terms of \( b \)?
   c. Using your answers to parts a and b, find the side length and area of the larger square.

27. **Determine** if the inequality \( 6x > 7x \) is never, sometimes, or always true. If it is sometimes true, identify the solution set.

28. **Use** the graph to find the theoretical probability of choosing each color.

![Favorite Color Graph](image)

29. Suppose \( d \) varies inversely with \( b \) and jointly with \( a \) and \( c \). Find the constant of variation when \( a = 4, b = 5, c = 2, \) and \( d = 8 \). Express the relationship between these quantities. What is \( d \) when \( a = 9, b = 15, \) and \( c = 6 \)?

30. Is \((x + 10)(x - 2)\) the correct factorization for \( x^2 - 8x - 20 \)? Explain.
Graphing Linear Inequalities

Linear inequalities in two variables can be graphed by hand or with a graphing calculator. Begin by following the process used to graph a linear equation. Then change the settings to shade the region of the coordinate system that makes the inequality true.

Graph the solution set of the inequality \( y > 2x - 7 \).

1. Enter the equation \( y = 2x - 7 \) into the \( Y = \) editor.

2. Graph the equation by pressing \( \text{Zoom} \) and selecting \( 6: \text{ZStandard} \).

   The line represents the boundary of the solution set of the inequality \( y > 2x - 7 \). The solution set of the inequality is either the region above or the region below the line \( y = 2x - 7 \).

   Use a test point to determine which region makes the inequality true.

   Choose a test point that is not on the graph of the line \( y = 2x - 7 \).

   The point \((0, 0)\) is a good test point because it does not fall on the boundary line.

   Substitute 0 for both \( x \) and \( y \) in the inequality. This substitution gives \( 0 > 2 \cdot 0 - 7 \), or \( 0 > -7 \), which is true.

   Since the point \((0, 0)\) satisfies the inequality, the solution set is the region that contains the point \((0, 0)\).

3. Shade the region above the line \( y = 2x - 7 \).

   To graph this region, press \( \text{Y=} \). Then press the \( \text{Y= editor} \), see the graphing calculator keystrokes in Lab 3 on page 305.

   Press \( \text{Enter} \) twice to choose the \( \backslash \) icon, which resembles a shaded region above a line.
(Pressing ENTER a third time allows you to choose the □ icon, which resembles a shaded region below a line.)

4. Press GRAPH to view the graph of the solution set.

All points in the shaded region are in the solution set of \( y > 2x - 7 \).

Note that the solution set does not include points on the line \( y = 2x - 7 \). The graph of the solution set of the inequality \( y \geq 2x - 7 \) does include points on the boundary line \( y = 2x - 7 \).

**Lab Practice**

Graph the solution set of each inequality.

a. \( y < 3x + 5 \); Is the point (1, 1) a solution of the inequality?

b. \( y \geq 2x - 5 \); Is the point (7, 2) a solution of the inequality?

c. \( y < -2x + 3 \); Is the point (0, 0) a solution of the inequality?
1. **Vocabulary** The equation of a line is \( y = mx + b \), where \( m \) is the slope of the line and \( b \) is the \( y \)-intercept.

Determine the slope and the \( y \)-intercept of each equation.

2. \( y = -\frac{1}{3}x - 5 \)

3. \( 2x + 2y = 6 \)

Graph each of the following inequalities on a number line.

4. \( y < 3 \)

5. \( x \geq -2 \)

A linear inequality is similar to a linear equation, except that a linear inequality has an inequality symbol instead of an equal sign. A solution of a linear inequality is any ordered pair that makes the inequality true.

You can evaluate an inequality with an ordered pair to find out if the ordered pair makes the inequality true and is a solution.

### Example 1 Determining Solutions of Inequalities

Determine if each ordered pair is a solution of the given inequality.

**a.** \((0, 4); \ y > 5x - 1\)

**SOLUTION**

\[ y > 5x - 1 \]

\[ 4 > 5(0) - 1 \] Evaluate the inequality for the point \((0, 4)\).

\[ 4 > -1 \] Simplify.

The inequality is true. The ordered pair \((0, 4)\) is a solution.

**b.** \((3, -3); \ y < -3x + 6\)

**SOLUTION**

\[ y < -3x + 6 \]

\[ -3 < -3(3) + 6 \] Evaluate the inequality for the point \((3, -3)\).

\[ -3 < -3 \] Simplify.

The inequality is not true because \(-3\) is not less than \(-3\). The ordered pair \((3, -3)\) is not a solution.

**c.** \((-4, 8); \ y \leq 9\)

**SOLUTION**

\[ y \leq 9 \]

\[ 8 \leq 9 \] Evaluate the inequality for the point \((-4, 8)\).

The inequality is true. The ordered pair \((-4, 8)\) is a solution.
**Exploration**  
**Graphing Inequalities**

a. Graph the equation \( y = x + 2 \) on a coordinate plane.

b. Test three points that lie above the graph of \( y = x + 2 \). Substitute the coordinates of each point for the \( x \)- and \( y \)-values in the inequality \( y < x + 2 \). If the statement is true, mark the point of the graph.

c. Test three points that lie below the graph of \( y = x + 2 \). Substitute the coordinates of each point for the \( x \)- and \( y \)-values in the inequality \( y < x + 2 \). If the statement is true, mark the point of the graph.

d. To graph the inequality \( y < x + 2 \), would you choose points above or below \( y = x + 2 \)?

e. Would you graph points above or below the graph of \( y = x - 3 \) to graph the inequality \( y > x - 3 \)?

A linear inequality describes a region of a coordinate plane called a half-plane. The boundary line for the region is the related equation.

To graph an inequality, begin by graphing the boundary line. Test points are helpful in deciding which half-plane makes the inequality true.

The boundary line is a dashed line when the inequality contains the symbol \(< \) or \(> \). The boundary line is a solid line when the inequality contains the symbol \(\leq \) or \(\geq \).

**Example 2**  
**Graphing Linear Inequalities without Technology**

Graph each inequality.

a. \( y \geq -\frac{3}{4}x - 3 \)

**SOLUTION**

Graph the boundary line \( y = -\frac{3}{4}x - 3 \) using a solid line because the inequality contains the symbol \(\geq \).

Next use an ordered pair as a test point to find which half-plane should be shaded on the coordinate plane.

Test \((0, 0)\).

\[
y \geq -\frac{3}{4}x - 3 \]

\[
0 \geq -\frac{3}{4}(0) - 3 \quad \text{Evaluate for } (0, 0).
\]

\[
0 \geq -3 \quad \text{Simplify.}
\]

The point \((0, 0)\) satisfies the inequality, so it is a solution. The half-plane that contains the point should be shaded.
Lesson 97

b. \( x < 4 \)

**SOLUTION**

Graph the boundary line \( x = 4 \) using a dashed line because the inequality contains the symbol \(<\).

Test \((0, 0)\).

\[ x < 4 \]

\[ 0 < 4 \quad \text{Evaluate for } (0, 0). \]

The point \((0, 0)\) satisfies the inequality, so it is a solution. The half-plane that contains the point should be shaded.

To graph an inequality using a graphing calculator, the inequality must first be solved for \( y \).

**Example 3**  **Graphing Linear Inequalities with Technology**

Graph each inequality using a graphing calculator.

**a.** \( 16x + 4y \leq 8 \)

**SOLUTION**

Solve for \( y \).

\[ 16x + 4y \leq 8 \]

\[ 4y \leq -16x + 8 \quad \text{Subtract } 16x \text{ from both sides.} \]

\[ \frac{4y}{4} \leq \frac{-16x}{4} + \frac{8}{4} \quad \text{Divide all three terms by } 4. \]

\[ y \leq -4x + 2 \quad \text{Simplify.} \]

Enter the inequality into your graphing calculator to view the graph.

**b.** \( 3x - y < -4 \)

**SOLUTION**

Solve for \( y \).

\[ 3x - y < -4 \]

\[ -y < -3x - 4 \quad \text{Subtract } 3x \text{ from both sides.} \]

\[ \frac{-y}{-1} > \frac{-3x}{-1} - \frac{4}{-1} \quad \text{Divide all three terms by } -1. \]

\[ y > 3x + 4 \quad \text{Simplify.} \]

Enter the inequality into your graphing calculator to view the graph.
Example 4 Writing a Linear Inequality Given the Graph

Write an inequality for the region graphed on each coordinate plane.

a.

SOLUTION

Determine the equation of the boundary line. It is a horizontal line that cuts through the $y$-axis at $-1$.

The equation of the boundary line is $y = -1$.

Then decide which inequality symbol to use for this graph.

The graph is shaded below the solid boundary line, so the inequality contains the $\leq$ symbol.

The inequality shown on the graph is $y \leq -1$.

b.

SOLUTION

Determine the equation of the boundary line. It has a $y$-intercept at $-4$ and a slope of $1$.

The equation of the boundary line is $y = x - 4$.

Then decide which inequality symbol to use for this graph.

The graph is shaded above the dashed boundary line, so the inequality contains the $>$ symbol.

The inequality shown on the graph is $y > x - 4$.

Example 5 Application: Carnival

Kia will attend a school carnival and she plans to spend no more than $12. Each game costs $2 and each item of food costs $3. Write and graph an inequality to describe the total cost of the carnival.
SOLUTION

Write the inequality that models the situation.

Cost of games plus cost of food is no more than 12

\[ 2x + 3y \leq 12 \]

Solve the inequality for \( y \).

\[ 2x + 3y \leq 12 \]
\[ 3y \leq -2x + 12 \quad \text{Subtract} \ 2x \ \text{from both sides.} \]
\[ y \leq -\frac{2}{3}x + 4 \quad \text{Divide all three terms by} \ 3 \ \text{and simplify.} \]

Graph the solutions.

Since Kia cannot buy a negative amount of games and food, use only Quadrant I. Graph the boundary line \( y = -\frac{2}{3}x + 4 \). Use a solid line for \( \leq \).

Shade below the line. Kia must buy whole numbers of games and food items. All the points on or below the line with whole-number coordinates are the different combinations of games and food Kia can buy.

Lesson Practice

Determine if each ordered pair is a solution of the given inequality.

\( (Ex\ 1) \)

a. \((2, 6)\); \( y > 3x - 2 \)
b. \((4, 1)\); \( y < -4x + 1 \)
c. \((-6, 2)\); \( y \leq 5 \)

Graph each inequality.

\( (Ex\ 2) \)
d. \(4x + 5y \geq -10 \)
e. \( x < 6 \)

Graph each inequality using a graphing calculator.

\( (Ex\ 3) \)
f. \(4x + 2y \leq 6 \)
g. \( y > 2x + 6 \)

Write an inequality for the region graphed on each coordinate plane.

\( (Ex\ 4) \)
h. i.
Nila has plans to attend the school bookfair and she wants to spend no more than $25. Each book series costs $15 and each book costs $5. Write an inequality to describe the total cost of the books Nila can buy and graph the inequality.

**Practice**

**Simplify.**

1. \[
\frac{30x^{-2}y^{12}}{6y^{-5}}
\]

2. \[
\sqrt{0.09q^2} + q\sqrt{0.04r}
\]

3. \[
\frac{16g^4}{2g + 3} - \frac{81}{2g + 3}
\]

4. Find the range of the data set that includes the ages of 9 members of a chess club: 23, 7, 44, 31, 18, 27, 35, 39, 66.

5. Find the product \((4x^2 + 8)(2x - 7)\) using the FOIL method.

6. Add \[
\frac{9}{9x - 36} + \frac{-24}{3x^2 - 48}
\]

7. Jim ran a total of \(\frac{x}{x^2 + 2x + 1}\) miles in the gym and \(\frac{x + 2}{x + 1}\) miles outside. How many more miles did he run inside?

8. Find the quotient of \((x^2 - 14x + 49) ÷ (x - 7)\).

**Solve and graph the inequality.**

9. \(13 \leq 2x + 7 < 15\)

10. \(\frac{|x|}{6} > 8\)

11. Determine if the inequality \(3x - 4x \geq 6 - x + 8\) is never, sometimes, or always true. If it is sometimes true, identify the solution set.

12. Determine if the ordered pair \((2, 6)\) is a solution of the inequality \(y > 3x - 2\).

13. Graph the function. \(y = x^2 + 2x - 24\)

14. Write What points on a graph of an inequality satisfy the inequality? Explain.

15. Generalize How do you know which half-plane to shade for the graph of a linear inequality?
**16. Multiple Choice** Which inequality represents the graph on the coordinate plane?

![Graph](image)

A. \( y \geq -\frac{2}{3}x + 2 \)

B. \( y \leq -\frac{2}{3}x + 2 \)

C. \( y \leq \frac{2}{3}x + 2 \)

D. \( y < -\frac{2}{3}x + 2 \)

**17. Football** Tickets for the school football game cost five dollars for adults and three dollars for students. In order to buy new helmets, at least $9000 worth of tickets must be sold. Write an inequality that describes the total number of tickets that must be sold in order to buy new helmets.

**18. Error Analysis** Two students find the vertex for \( y = x^2 - 6x + 19 \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{b}{2a} = \frac{6}{2} = 3) The vertex is (3, 0).</td>
<td>(-\frac{b}{2a} = \frac{6}{2} = 3) (3^2 - 6(3) + 19 = 10) The vertex is (3, 10).</td>
</tr>
</tbody>
</table>

**19. Geometry** The area of a rectangle is 48 square inches. The length is three times the width. Find the width of the rectangle by finding the positive zero of the function \( y = 3w^2 - 48 \).

**20. Multi-Step** The height \( y \) of a golf ball in feet is given by the function \( y = -16x^2 + 49x \).

a. What is the \( y \)-intercept?

b. What does this \( y \)-intercept represent?

c. What answer does the equation give for the height of the ball after 5 seconds?

d. What does that height mean?

**21. Commuting** Jeff traveled \( \frac{1}{2x^2 - 4x} \) miles for his job on Monday and \( \frac{1}{x^2 - 2x} \) miles for his job on Tuesday. How many more miles did he travel on Tuesday?

**22. Multiple Choice** Add \( \frac{3y + 2}{y + z} + \frac{4}{2y + 2z} \).

A. \( \frac{3y + 4}{y + z} \)

B. \( \frac{y + 4}{y + z} \)

C. \( \frac{6y}{4mn} \)

D. \( \frac{3y - 4}{y - z} \)
23. **Verify** Show that $-11$ is a solution to the inequality $|3x| - 2 = 31$.

24. **Multiple Choice** Solve $4|x - 8| = 12$.

   A $\{-5, 5\}$  
   B $\{11, -11\}$  
   C $\{5, 11\}$  
   D $\{-20, 20\}$

25. **Bike Riding** Ron rode his bike for $\frac{10}{45x^2 + 4x - 1}$ minutes to get to his grandmother’s house that was $\frac{1}{45x - 5} + \frac{2x}{25x + 5}$ miles away. Find his rate in miles per minute.

26. **Carpentry** A carpenter uses a measuring tape with an accuracy of $\pm \frac{1}{32}$ inches. He measures the height of a bookshelf to be $95\frac{5}{8}$ inches. Solve the inequality $\left| x - 95\frac{5}{8} \right| \leq \frac{1}{32}$ to find the range of the height of the bookshelf.

27. **Generalize** How are the number of zeros of a function related to the location of the vertex of the function’s parabola?

28. **Probability** The probability of winning a certain game is $\frac{2x^4y^2}{15x^3}$. The probability of winning a different game is $\frac{5x^2y}{8x^3y^2}$. What is the probability of winning both games?

29. **Rate** An orange juice machine squeezes juice out of $x^2 + 30x$ oranges every hour. How much time in days will it take to squeeze 3000 oranges?

30. **Multi-Step** A circle has a radius of $x$. Another circle has a radius of $3x$.

   a. Write equations for the areas of both circles.
   
   b. Graph both functions in the same coordinate plane.
   
   c. Compare the graphs.
Warm Up

1. **Vocabulary** A \_\_\_\_\_\_\_\_\_\_\_\_\_ is an \(x\)-value for the function where \(f(x) = 0\).

Factor.

2. \(x^2 + 3x - 88\)  
3. \(6x^2 - 7x - 5\)  
4. \(4x^2 + 28x + 49\)  
5. \(12x^2 - 27\)

New Concepts

A **root of an equation** is the solution to an equation. A quadratic equation can have zero, one, or two roots. The roots of a quadratic equation are the \(x\)-intercepts, or zeros, of the related quadratic function.

To find the roots of a quadratic equation, set the equation equal to 0. If the quadratic expression can be factored, the equation can be solved using the **Zero Product Property**.

### Zero Product Property

If the product of two quantities equals zero, at least one of the quantities equals zero.

**Example 1** Using the Zero Product Property

Solve.

\((x - 4)(x + 5) = 0\)

**SOLUTION**

By the Zero Product Property, one or both of these factors must be equal to 0. To find the solutions, set each factor equal to zero and solve.

\(x - 4 = 0\)  \(x + 5 = 0\)  
\(x = 4\)  \(x = -5\)  
Set each factor equal to zero.  
Solve each equation for \(x\).

**Check** Substitute each solution into the original equation to show it is true.

\((x - 4)(x + 5) = 0\)  
\((4 - 4)(4 + 5) = 0\)  
\(0 \cdot 9 = 0\)  
\(0 = 0\)  \(\checkmark\)

\((-5 - 4)(-5 + 5) = 0\)  
\((-9) \cdot 0 = 0\)  
\(0 = 0\)  \(\checkmark\)

The solution set is \(-5, 4\).
Example 2  Solving Quadratic Equations by Factoring

Find the roots.

a. \(x^2 + 2x = 8\)

\textbf{SOLUTION}

\[x^2 + 2x = 8\]
\[x^2 + 2x - 8 = 0\]  Set the equation equal to 0.
\[(x + 4)(x - 2) = 0\]  Factor.

Use the Zero Product Property to solve the equation.

\[x + 4 = 0\]
\[x = -4\]
\[x - 2 = 0\]
\[x = 2\]

\textbf{Check}

\[x^2 + 2x = 8\]
\[(-4)^2 + 2(-4) \neq 8\]
\[16 - 8 \neq 8\]
\[8 = 8\]

\[x^2 + 2x = 8\]
\[(2)^2 + 2(2) = 8\]
\[4 + 4 = 8\]
\[8 = 8\]

The roots are -4 and 2.

b. \(7x^2 - 6 = 19x\)

\textbf{SOLUTION}

\[7x^2 - 6 = 19x\]
\[7x^2 - 19x - 6 = 0\]  Set the equation equal to 0.
\[(7x + 2)(x - 3) = 0\]  Factor.

\[7x + 2 = 0\]
\[x = -2\]
\[x - 3 = 0\]
\[x = 3\]

\textbf{Check}

\[7x^2 - 6 = 19x\]
\[7\left(-\frac{2}{7}\right)^2 - 6 \neq 19\left(-\frac{2}{7}\right)\]
\[\left(\frac{4}{7}\right) - 6 \neq -\frac{38}{7}\]
\[-\frac{38}{7} = -\frac{38}{7}\]

The roots are \(-\frac{2}{7}\) and 3.
Example 3 Finding the Roots by Factoring Out the GCF

Find the roots.

\[ 20 - 2x^2 = 70 - 20x \]

**SOLUTION**

\[ 2x^2 - 20x + 50 = 0 \]  
Set the equation equal to zero.

\[ 2(x^2 - 10x + 25) = 0 \]  
Factor out the GCF.

\[ 2(x - 5)(x - 5) = 0 \]  
Factor the trinomial expression.

Disregard the factor of 2, since it can never equal 0.

The factor \((x - 5)\) appears twice, but it only needs to be set to equal zero once.

\[ x - 5 = 0 \]
\[ x = 5 \]

**Check**

\[ 20 - 2x^2 = 70 - 20x \]
\[ 20 - 2(5)^2 = 70 - 20(5) \]
\[ 20 - 2(25) = 70 - 100 \]
\[ 20 - 50 = -30 \]
\[ -30 = -30 \quad \checkmark \]

The root is 5.

Finding the values of \(x\) that satisfy the quadratic equation is another way of finding the roots of a quadratic equation.

Example 4 Application: Gardening

The area of a rectangular garden is 51 square yards. The length is 14 yards more than the width. What are the length and width of the garden?

**SOLUTION**

Let \(w\) be the width and \(w + 14\) be the length.

\[ A = lw \]  
Area formula

\[ 51 = (w + 14)w \]  
Substitute known values into the equation.

\[ 51 = w^2 + 14w \]  
Distribute.

\[ 0 = w^2 + 14w - 51 \]  
Write the equation into standard form.

\[ 0 = (w + 17)(w - 3) \]  
Factor.

\[ w + 17 = 0 \quad w - 3 = 0 \]  
Use the Zero Product Property.

\[ w = -17 \quad w = 3 \]  
Solve.

Because the width must be a positive number, the only possible solution is 3 yards. Since the width is 3 yards, the length is \(w + 14\). So the length is 3 + 14, which is 17 yards.
Example 5 Solving Quadratic Equations with Missing Terms

Solve.

a. \(18x^2 = 8x\)

**SOLUTION**

\[18x^2 - 8x = 0\]  Set the equation equal to zero.

\[2x(9x - 4) = 0\]  Factor out the GCF.

\[2x = 0 \quad 9x - 4 = 0\]  Set each factor equal to zero.

\[x = 0 \quad x = \frac{4}{9}\]  Solve each equation for \(x\).

**Check**

\[18x^2 = 8x\]

\[18 \cdot (0)^2 \div 8(0)\]

\[0 = 0 \quad \checkmark\]

\[18 \left(\frac{4}{9}\right)^2 \div 8\left(\frac{4}{9}\right)\]

\[18 \left(\frac{16}{81}\right) \div 8\left(\frac{4}{9}\right)\]

\[\frac{32}{9} = \frac{32}{9} \quad \checkmark\]

The solution set is \(\{0, \frac{4}{9}\}\).

b. \(4x^2 - 25 = 0\)

**SOLUTION**

\[4x^2 - 25 = 0\]  Set the equation equal to 0.

\[(2x - 5)(2x + 5) = 0\]  Factor.

\[2x - 5 = 0 \quad 2x + 5 = 0\]  Set each factor equal to zero.

\[2x = 5 \quad 2x = -5\]  Solve each equation for \(x\).

\[x = 2.5 \quad x = -2.5\]

**Check**

\[4x^2 - 25 = 0\]

\[4(2.5)^2 - 25 \div 0\]

\[4 \cdot (-2.5)^2 - 25 \div 0\]

\[4(6.25) - 25 \div 0\]

\[25 - 25 \div 0\]

\[0 = 0 \quad \checkmark\]

The solution set is \((-2.5, 2.5)\).
Solve and graph the inequality.

1. \(11 < 2(x + 5) < 20\)
2. \(|x| + 1.5 \leq 7.6\)

3. Determine whether the polynomial \(-121 + 9x^2\) is a perfect-square trinomial or a difference of two squares. Then factor the polynomial.

4. Graph the function \(y = 2x^2 + 8x + 6\).

5. The number of Apples \(A\) and Oranges \(O\) grown in a certain fruit orchard can be modeled by the given expressions where \(x\) is the number of years since the trees were planted. Find a model that represents the total number of apples and oranges grown in this orchard.

\[
A = 15x^3 + 17x - 20 \\
O = 20x^3 + 11x - 4
\]

6. Write an equation for a line that passes through \((1, 2)\) and is perpendicular to \(y = \frac{-3}{4}x + 2\frac{3}{4}\).

7. Solve the equation \(\frac{4|x|}{9} + 3 = 11\) and graph the solution.

Simplify.

8. \(\frac{3x + 6}{7x - 7}\)
9. \(\frac{5x + 10}{14x - 14}\)

*10. Determine if the ordered pair \((5, 5)\) is a solution of the inequality \(y < -5x + 4\).

*11. Write Explain the Zero Product Property in your own words.
12. **Justify** What property allows you to use the following step when solving an equation?

\[(x + 4)(x + 5) = 0\]
\[x + 4 = 0 \quad x + 5 = 0\]

13. **Multiple Choice** What is the solution set of \(0 = (3x - 5)(x + 2)\)?
A \[\left\{\frac{5}{3}, -2\right\}\]
B \[\left\{\frac{5}{3}, 2\right\}\]
C \[\left\{-\frac{5}{3}, -2\right\}\]
D \[\left\{-\frac{5}{3}, 2\right\}\]

14. **Ages** A girl is 27 years younger than her mother. Her mother is \(m\) years old. The product of their ages is 324. How old is each person?

15. **Multi-Step** Seve plans to go shopping for new jeans and shorts. She plans to spend no more than $70. Each pair of jeans costs $20 and each pair of shorts costs $10.
   a. Write an inequality that describes this situation.
   b. Graph the inequality.
   c. If Seve wants to spend exactly $70, what is a possible number of each she can spend her money on?

16. **Geometry** The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. The sides of a triangle are labeled \(4x\) inches, \(2y\) inches, and 8 inches. James wrote an inequality that satisfies the Triangle Inequality Theorem. He wrote the inequality \(4x + 2y > 8\). Use a graphing calculator to graph the inequality.

17. Solve \((x + 4)(x - 9) = 0\).

18. **Error Analysis** Students were asked to write an inequality that results in a dashed horizontal boundary line. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th></th>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y &gt; -3)</td>
<td>(y \geq 4)</td>
<td></td>
</tr>
</tbody>
</table>

19. **Horseback Riding** It took Joe \(\frac{2x - 10}{3x^2 - 15x} / 3x\) minutes to ride his horse to Darrell’s house that was \(\frac{3x^2 - 15x}{3x}\) miles away. Find his rate in miles per minute.

20. **Measurement** The area of a triangle can be expressed as \(4x^2 - 2x - 6\) square meters. The height of the triangle is \(x + 1\) meters. Find the length of the base of the triangle.

21. **Art** Michael bought a rectangular painting from a local artist. The area of the painting was \((20x + 5 + x^3)\) square inches. The width was \((x - 5)\) inches. What was the length?

22. **Analyze** Should you find the LCD when multiplying or dividing rational fractions?
23. **Multi-Step**  Erika and Casey started a new walking program. They walked \( \frac{4x}{3x+9} \) miles Thursday and \( \frac{16}{x^2+12x+27} \) miles Friday.
   a. What is the total distance that they walked?
   b. If their rate was \( \frac{4x}{x+3} \) miles per hour, how much time did it take them to walk on Thursday and Friday?

*24. **Physics**  A ball is dropped from 100 feet in the air. What is its height after 2 seconds? Use \( h = -16t^2 + vt + s \). (Hint: Its initial velocity is 0 feet per second.)

*25. **Error Analysis**  Two students find the zeros of the function \( y = x^2 - 8x - 33 \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^2 - 8x - 33 )</td>
<td>( 0 = x^2 - 8x - 33 )</td>
</tr>
<tr>
<td>( y = (0)^2 - 8(0) - 33 )</td>
<td>( 0 = (x - 11)(x + 3) )</td>
</tr>
<tr>
<td>( y = -33 )</td>
<td>( x - 11 = 0 ) or ( x + 3 = 0 )</td>
</tr>
<tr>
<td>( (0, -33) )</td>
<td>( x = 11 ) or ( x = -3 )</td>
</tr>
<tr>
<td>( 11 ) and ( -3 )</td>
<td></td>
</tr>
</tbody>
</table>

26. **Multi-Step**  Hideyo has a picture frame that measures 20 centimeters by 26 centimeters. The frame is 1.5 centimeters wide.
   a. Find the length and width of the picture area.
   b. Find the length of the diagonal of the picture area to the nearest tenth of a centimeter.

27. **Profit**  A business sold \( x^2 + 6x + 5 \) items. The profit for each item sold is \( \frac{x^2}{100} \) dollars.
   a. What is the profit in terms of \( x \)?
   b. What is the profit (in dollars) if \( x = 50 \)?

28. **Multi-Step**  Javed has a garden with an area of 36 square feet. The width of his garden is 9 feet less than the length. What are the dimensions of his garden?
   a. Write a formula to find the dimensions of the garden and describe how you will solve it.
   b. What are the dimensions of the garden?

29. **Generalize**  Describe the steps for subtracting \( \frac{x - 6}{x + 5} \) from \( \frac{x^2 - x - 30}{x + 5} \).

30. The width of a rectangle is represented by the expression \( 4x - 6 \) and the length \( 8x + 7 \). Would the area of the rectangle be correctly expressed as \( 32x^2 + 20x - 42 \)? If not, what is the correct area?
Warm Up

1. **Vocabulary** The denominator of a ________ contains a variable. The value of the variable cannot make the denominator equal to zero.

2. Find the LCM.
   - $7x^2y$ and $3xy^3$
   - $(x + 3)$ and $(2x - 1)$
   - $(3x - 6)$ and $(9x^2 - 18x)$
   - $(14x - 7y)$ and $(10x - 5y)$

New Concepts

A **rational equation** is an equation containing at least one rational expression. There are two ways to solve a rational equation: using cross products or using the least common denominator.

Either way may lead to an **extraneous solution**; that is, a solution that does not satisfy the original equation. The solution may satisfy a transformed equation, but make a denominator in the original equation equal 0. If an answer is extraneous, eliminate it from the solution set.

If a rational equation is a proportion, it can be solved using cross products.

Example 1  **Solving a Rational Proportion**

Solve each equation.

**(a)** $\frac{3}{x} = \frac{5}{x - 6}$

**SOLUTION**

\[
\begin{align*}
3 & = \frac{5}{x - 6} \\
3(x - 6) & = 5x \\
3x - 18 & = 5x \\
-18 & = 2x \\
-9 & = x
\end{align*}
\]

**Check** Verify that the solution is not extraneous.

\[
\begin{align*}
\frac{3}{x} & = \frac{5}{x - 6} \\
\frac{3}{-9} & = \frac{5}{-9 - 6} \\
\frac{3}{-9} & = \frac{5}{-15} \\
\frac{-1}{3} & = \frac{-1}{3}
\end{align*}
\]

The solution is $x = -9$.  ✔
b. \( \frac{x}{4} = \frac{3}{x - 1} \)

**SOLUTION**

\[
\frac{x}{4} = \frac{3}{x - 1}
\]

\[
x(x - 1) = 12 \quad \text{Use cross products.}
\]

\[
x^2 - x = 12 \quad \text{Distribute} \ x \text{ over} \ (x - 1).
\]

\[
x^2 - x - 12 = 0 \quad \text{Subtract} \ 12 \text{ from both sides.}
\]

\[
(x - 4)(x + 3) = 0 \quad \text{Factor.}
\]

\[
x = 4 \text{ or } x = -3 \quad \text{Use the Zero Product Property to solve.}
\]

**Check** Verify that the solution is not extraneous.

\[
\frac{x}{4} = \frac{3}{x - 1}; \ x = 4 \quad \text{or} \quad \frac{x}{4} = \frac{3}{x - 1}; \ x = -3
\]

\[
\frac{4}{4} = \frac{3}{4 - 1}
\]

Substitute. \[
\frac{-3}{4} \neq \frac{3}{-3 - 1}
\]

\[
1 = 1 \quad \checkmark
\]

Simplify each fraction. \[
\frac{-3}{4} = \frac{-3}{4} \quad \checkmark
\]

The solution set is \( \{4, -3\} \).

If a rational equation includes a sum or difference, find the LCD of all the terms to solve.

**Example 2** Using the LCD to Solve Addition Equations

Solve \( \frac{3}{x} + \frac{16}{2x} = 11 \).

**SOLUTION**

\[
\frac{3}{x} + \frac{16}{2x} = 11
\]

The LCD of the denominators is \( 2x \).

\[
(2x)\frac{3}{x} + (2x)\frac{16}{2x} = (2x)11 \quad \text{Multiply each term by the LCD.}
\]

\[
6 + 16 = 22x \quad \text{Simplify each term.}
\]

\[
22 = 22x \quad \text{Add.}
\]

\[
x = 1 \quad \text{Divide both sides by} \ 22.
\]

**Check** Verify that the solution is not extraneous.

\[
\frac{3}{x} + \frac{16}{2x} = 11
\]

\[
\frac{3}{1} + \frac{16}{2(1)} = 11 \quad \text{Substitute} \ 1 \text{ for} \ x \text{ in the original equation.}
\]

\[
11 = 11 \quad \checkmark \quad \text{Simplify.}
\]

The solution is \( x = 1 \).
Example 3  Using the LCD to Solve Subtraction Equations

Solve \( \frac{3}{x - 1} - \frac{2}{x} = \frac{5}{2x} \).

**SOLUTION**

\[
\frac{3}{x - 1} - \frac{2}{x} = \frac{5}{2x}
\]

The LCD is \( 2x(x - 1) \).

\[
2x(x - 1) \frac{3}{x - 1} - 2x(x - 1) \frac{2}{x} = 2x(x - 1) \frac{5}{2x}
\]

Multiply each term by the LCD.

\[
6x - 4(x - 1) = 5(x - 1)
\]

Simplify each term.

\[
6x - 4x + 4 = 5x - 5
\]

Use the Distributive Property.

\[
2x + 4 = 5x - 5
\]

Collect like terms.

\[
4 = 3x - 5
\]

Subtract 2x from both sides.

\[
9 = 3x
\]

Add 5 to both sides.

\[
3 = x
\]

Divide both sides by 3.

**Check** Verify that the solution is not extraneous.

\[
\frac{3}{x - 1} - \frac{2}{x} = \frac{5}{2x}
\]

\[
\frac{3}{3 - 1} - \frac{2}{3} = \frac{5}{2(3)}
\]

Substitute 3 for \( x \) in the original equation.

\[
\frac{5}{6} = \frac{5}{6}
\]

Simplify.

The solution is \( x = 3 \).

Example 4  Checking for Extraneous Solutions

Solve the equation.

\[
\frac{x - 1}{x - 2} = \frac{x + 9}{2x - 4}
\]

**SOLUTION**

\[
\frac{x - 1}{x - 2} = \frac{x + 9}{2x - 4}
\]

Use cross products.

\[
(x - 1)(2x - 4) = (x - 2)(x + 9)
\]

Multiply.

\[
2x^2 - 6x + 4 = x^2 + 7x - 18
\]

Subtract \( x^2 \), 7x, and -18 from both sides.

\[
x^2 - 13x + 22 = 0
\]

Factor.

\[
(x - 11)(x - 2) = 0
\]

Use the Zero Product Property to solve.

\[
x = 11 \text{ or } x = 2
\]
Check Verify that the solutions are not extraneous.

\[
\frac{x - 1}{x - 2} = \frac{x + 9}{2x - 4}; \quad x = 11 \quad \text{or} \quad \frac{x - 1}{x - 2} = \frac{x + 9}{2x - 4}; \quad x = 2
\]

\[
\frac{11 - 1}{11 - 2} = \frac{11 + 9}{2(11) - 4}
\]

Substitute.

\[
\frac{2 - 1}{2 - 2} = \frac{2 + 9}{2(2) - 4}
\]

Simplify.

\[
\frac{10}{9} = \frac{10}{9} \quad \checkmark
\]

The solution is \(x = 11\).

Example 5 Application: Painting

It takes Samuel 7 hours to paint a house. It takes Jake 5 hours to paint the same house. How long will it take them if they work together?

SOLUTION

Understand The answer will be the number of hours \(h\) it takes for Samuel and Jake to paint the house.

Samuel can paint the house in 7 hours, so he can paint \(\frac{1}{7}\) of the house per hour.

Jake can paint the house in 5 hours, so he can paint \(\frac{1}{5}\) of the house per hour.

Plan The part of the house Samuel paints plus the part of the house Jake paints equals the complete job. Samuel’s rate times the number of hours worked plus Jake’s rate times the number of hours worked will give the complete time it will take them to paint the house. Let \(h\) represent the number of hours worked.

\[(\text{Samuel’s rate})h + (\text{Jake’s rate})h = \text{complete job}\]

\[
\frac{1}{7}h + \frac{1}{5}h = 1
\]

Solve

\[
\frac{1}{7}h + \frac{1}{5}h = 1
\]

\[
(35)\frac{1}{7}h + (35)\frac{1}{5}h = (35)1 \quad \text{Multiply each term by the LCD, 35.}
\]

\[
5h + 7h = 35 \quad \text{Simplify each term.}
\]

\[
h = \frac{35}{12} \quad \text{Combine like terms; and divide both sides by 12.}
\]

Together, they can paint the house in \(2\frac{11}{12}\) hours.
Lesson Practice

Solve each equation.

a. \( \frac{6}{x} = \frac{7}{x - 1} \)  

b. \( \frac{2}{x + 4} = \frac{x}{6} \)  

c. \( \frac{12}{2x} + \frac{16}{4x} = 5 \)  

d. \( \frac{4}{x - 2} - \frac{2}{x} = \frac{1}{3x} \)  

e. \( \frac{x + 5}{x + 4} = \frac{x - 2}{2x + 8} \)  

f. (Lawn Care) It takes John 2 hours to mow the yard. Sarah can do it in 3 hours. How long will it take them if they mow the yard together?

Practice

Distributed and Integrated

Solve.

1. \( \frac{4}{x} = \frac{8}{x + 4} \)  

2. \( (x - 13)(x + 22) = 0 \)  

3. (Physics) A student is biking to a friend’s house. He bikes at 10 miles per hour. The friend lives 20 miles away, give or take 2 miles. What are the minimum and maximum times it will take him to get there?

4. Verify Without graphing, show that the point \((2, -6)\) lies on the graph of \(y = x^2 + x - 12\).

5. Simplify: \( \frac{4x}{12x - 60} + \frac{1}{4x - 16} \)  

6. Factor \( x + 4x^2 - 5 \).

7. Larry weighed 180 pounds. He has lost 2 pounds a month for \(x\) months. Write a linear equation to model his weight after 8 months.

8. Write What is an extraneous solution?

9. Find the LCD of \( \frac{2x}{2x^2 - 72} - \frac{12}{x^2 + 13x + 42} \).

10. (Population) The function \( y = -0.0003x^2 + 0.03x + 1.3 \) shows the population of Philadelphia County between the years 1900 and 1990, where \(x\) is the number of years after 1900 and \(y\) is the population for that year in millions of people. Find the vertex of the parabola that represents the function. What does it represent in terms of the scenario?
11. **Multi-Step** The coordinates of three friends’ houses on a city map are \(P(3, 3)\), \(Q(5, 9)\), and \(R(11, 3)\). The friends plan to meet at the point that is half-way between \(Q\) and the midpoint of \(PR\).
   a. Find the midpoint of \(PR\).
   b. Find the coordinates of the point where the friends plan to meet.

12. Find the quotient of \(18x^2 - 120 + 6x^3 \div (x - 2)\).

13. What is the ratio of the volume of a cube with side lengths of 5 to the volume of a cube with side lengths of 3?

*14. Formulate* How can you quickly tell if a possible solution is extraneous using the denominators in the original equation?

*15. Multiple Choice* Which of the following is an extraneous solution to the equation \(\frac{x^2}{x - 1} + \frac{4x^2 - 20x}{(x - 1)(x - 5)} = \frac{10}{2x - 2}\)?
   A \(x = -1\)  
   B \(x = 0\)  
   C \(x = 1\)  
   D \(x = 5\)

*16. Housekeeping* It takes a man 8 hours to clean the house. His friend can clean it in 6 hours. How long will it take them if they clean together?

*17. Error Analysis* Two students solve the equation \(0 = (x - 5)(x + 11)\). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
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</thead>
<tbody>
<tr>
<td>(x - 5 = 0)</td>
<td>(x - 5 = 0)</td>
</tr>
<tr>
<td>(x + 11 = 0)</td>
<td>(x + 11 = 0)</td>
</tr>
<tr>
<td>(x = 5)</td>
<td>(x = -5)</td>
</tr>
<tr>
<td>(x = -11)</td>
<td>(x = 11)</td>
</tr>
</tbody>
</table>

18. **Geometry** The area of a triangle is 24 square centimeters. The height is four more than two times the base. Find the base and height of the triangle.

*19. Multi-Step* The length of a lawn is twice the width. The lawnmower cuts 2-foot strips. One strip along the length and width has already been cut.

![Diagram of a lawn with dimensions](image)

a. Write expressions for the length and width of the area left to be cut.
   b. The area left to be cut is 144 square feet. Find the width of the yard.
   c. What is the length of the yard?

20. **Generalize** How do you know when there is no solution to an absolute-value inequality?

21. **Architecture** Mia is designing a rectangular city hall. She accidentally spills water on her newly revised sketches. She is only able to determine the area, which is \((x^2 - 15x + 56)\) square feet, and the length, which is \((x - 7)\) feet. What is the width?
22. **Multiple Choice** What are the zeros for the function \( y = 4x^2 + 28x - 72 \)?
   - A 0, 4
   - B 0, -4
   - C 2, -9
   - D 2, -76

23. **Band** The school band is performing a music concert. Tickets cost $3 for adults and $2 for students. In order to cover expenses, at least $200 worth of tickets must be sold. Write an inequality that describes the graph of this situation.

24. **Measurement** Jesse bought a new glass window to go in his front room. The area of the window is \((2 + 3x^3 - 8x)\) square inches. The width is \((x + 4)\) inches. What is the length?

25. **Error Analysis** Students were asked to write an inequality that results in a solid, vertical boundary line. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x \leq 6 )</td>
<td>( x &lt; -2 )</td>
</tr>
</tbody>
</table>

26. Graph the inequality \( 4x + 5y \geq -7 \).

27. **Multi-Step** Pedro biked 6 miles on dirt trails and 12 miles on the street. His biking rate on the dirt trails was 25% of what it was on the street.
   a. Write a simplified expression for Pedro’s total biking time.
   b. **Analyze** Explain how finding the simplified expression would change if Pedro’s biking rate on the trails was 50% of what it was on the streets.

28. Graph the function \( y - 3 = -x^2 + 3 \).

29. A square has a side length \( s \). The square grows larger and has a new area of \( s^2 + 16s + 64 \). What is the new side length?

30. Write an equation where \( j \) is inversely proportional to \( m \) and \( n \) and directly proportional to \( p \) and \( q \).
Solving Quadratic Equations by Graphing

**Warm Up**

1. **Vocabulary** The U-shaped curve that results from graphing a quadratic function is called a(n) _______

Evaluate each expression for the given values.

2. \(3(x - y)^2 - 4y^2\) for \(x = -5\) and \(y = -2\)
3. \(-x^2 - 3xy + y\) for \(x = 3\) and \(y = -1\)

Determine the direction that the parabola opens.

4. \(f(x) = 3x^2 + x - 4\)
5. \(f(x) = -2x^2 + x + 1\)

**New Concepts**

The solution(s) of a quadratic equation, \(0 = ax^2 + bx + c\), can be found by graphing the related function, \(f(x) = ax^2 + bx + c\). The U-shaped graph of a quadratic function is called a parabola. The solutions of the equation are called roots and can be found by determining the \(x\)-intercepts or zeros of the quadratic function. These zeros can be found by graphing the related function to see where the parabola intersects the \(x\)-axis.

**Graphical Solutions**

<table>
<thead>
<tr>
<th>One Real Solution</th>
<th>Two Real Solutions</th>
<th>No Real Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The graph intersects the (x)-axis at the vertex.</td>
<td>The graph intersects the (x)-axis at two distinct points.</td>
<td>The graph does not intersect the (x)-axis.</td>
</tr>
</tbody>
</table>

**Math Language**

The same function is described by \(y = 3x^2 - 5\) and \(f(x) = 3x^2 - 5\).

The function notation for \(y\) is \(f(x)\). It is read, “\(f\) of \(x\).”

**Online Connection**

www.SaxonMathResources.com
Example 1  Solving Quadratic Equations by Graphing

Solve each equation by graphing the related function.

a. \( x^2 - 36 = 0 \)

SOLUTION

Step 1: Find the axis of symmetry.

\[
x = \frac{-b}{2a}
\]

Use the formula.

\[
x = \frac{0}{2(1)} = 0
\]

Substitute values for \( a \) and \( b \).

The axis of symmetry is \( x = 0 \).

Step 2: Find the vertex.

\[
f(x) = x^2 - 36
\]

\[
f(0) = (0)^2 - 36
\]

Evaluate the function for \( x = 0 \) to find the vertex.

The vertex is \((0, -36)\).

Step 3: Find the \( y \)-intercept.

The \( y \)-intercept is \( c \), or \(-36\).

Step 4: Find two more points that are not on the axis of symmetry.

\[
f(5) = 5^2 - 36 \quad f(7) = 7^2 - 36
\]

\[
(5, -11) \quad (7, 13)
\]

Step 5: Graph.

Graph the axis of symmetry \( x = 0 \), the vertex and the \( y \)-intercept, both at coordinate \((0, -36)\). Reflect the points \((5, -11)\) and \((7, 13)\) over the axis of symmetry and graph the points \((-5, -11)\) and \((-7, 13)\). Connect the points with a smooth curve.

From the graph, the \( x \)-intercepts appear to be 6 and \(-6\).

Check  Substitute the values for \( x \) in the original equation.

\[
x^2 - 36 = 0; \quad x = 6 \quad x^2 - 36 = 0; \quad x = -6
\]

\[
(6)^2 - 36 \div 0 \quad (-6)^2 - 36 \div 0
\]

\[
36 - 36 \div 0 \quad 36 - 36 \div 0
\]

\[
0 = 0 \quad \checkmark \quad 0 = 0 \quad \checkmark
\]

The solutions are 6 and \(-6\).
Lesson 100

b. \(-x^2 - 2 = 0\)

**SOLUTION**

Graph the related function \(f(x) = -x^2 - 2\).

axis of symmetry: \(x = 0\)

vertex: \((0, -2)\)

\(y\)-intercept: \((0, -2)\)

two additional points: \((1, -3)\) and \((3, -11)\)

Reflect these two points across the axis of symmetry and connect them with a smooth curve.

From the graph, it can be seen that there is no \(x\)-intercept because the graph does not intersect the \(x\)-axis.

There is no real-number solution.

c. \(x^2 + 16 = 8x\)

**SOLUTION**

Write the equation in standard form.

\(x^2 - 8x + 16 = 0\)

Graph the related function \(f(x) = x^2 - 8x + 16\).

axis of symmetry: \(x = 4\)

vertex: \((4, 0)\)

\(y\)-intercept: \((0, 16)\)

two additional points: \((2, 4)\) and \((3, 1)\)

Reflect these two points across the axis of symmetry and connect them with a smooth curve.

From the graph, the \(x\)-intercept appears to be 4.

**Check** Substitute 4 for \(x\) in the original equation.

\(x^2 - 8x + 16 = 0; \ x = 4\)

\((4)^2 - 8(4) + 16 \quad \checkmark 0\)

\(16 - 32 + 16 = 0\)

\(0 = 0 \ \checkmark\)

The solution is 4.

---

**Caution**

When a parabola does not cross the \(x\)-axis, there is no real-number solution to the quadratic equation.
Example 2  Solving Quadratic Equations Using a Graphing Calculator

Solve each equation by graphing the related function on a graphing calculator.

a. \(-6x - 9 = x^2\)

**SOLUTION**
Write the equation in standard form.
\(-x^2 - 6x - 9 = 0\)
Graph the related function
\(f(x) = -x^2 - 6x - 9\).
The graph appears to have an \(x\)-intercept at \(-3\).
Use the Table function to determine the zeros of this function.
The solution is \(-3\).

b. \(-6x = -x^2 - 13\)

**SOLUTION**
Write the equation in standard form.
\(x^2 - 6x + 13 = 0\).
Graph the related function \(f(x) = x^2 - 6x + 13\).
The graph opens upward and does not intersect the \(x\)-axis.
There is no solution.

c. \(-3x^2 + 5x = -7\)
Round to the nearest tenth.

**SOLUTION**
Write the equation in standard form.
\(-3x^2 + 5x + 7 = 0\)
Graph the related function
\(f(x) = -3x^2 + 5x + 7\).
The graph appears to have \(x\)-intercepts at 3 and \(-1\).
Use the Zero function to determine the zeros of this function. Round to the nearest tenth.
The solutions are \(x = 2.6\) and \(-0.9\).

**Hint**
For help with graphing quadratic functions, see Graphing Calculator Lab 8: Characteristics of Parabolas on p. 583.
Example 3  Application: Physics

Gill drops a baseball from the top of a platform 64 feet off the ground. The height of the baseball is described by the quadratic equation \( h = -16t^2 + 64 \), where \( h \) is the height in feet and \( t \) is the time in seconds. Find the time \( t \) when the ball hits the ground.

**SOLUTION**

Graph the related function \( h(t) = -16t^2 + 64 \) on a graphing calculator.

Height \( h \) is zero when the ball hits the ground. Use the Zero function of the graphing calculator to determine the zeros of this function.

There are two zeros for the given parabola: \( t = 2 \) and \( t = -2 \). Only values greater than or equal to zero are considered.

So, \( t = 2 \) is the only solution.

The baseball hits the ground in 2 seconds.

**Lesson Practice**

Solve each equation by graphing the related function.

a. \( 3x^2 - 147 = 0 \)

b. \( 5x^2 + 6 = 0 \)

c. \( x^2 - 10x + 25 = 0 \)

d. \( x^2 + 64 = 16x \)

e. \( x^2 + 4 = 2x \)

Solve each equation by graphing the related function on a graphing calculator.

f. Round to the nearest tenth: \( -7x^2 + 3x = -7 \).

g. Marcus shot an arrow while standing on a platform. The path of its movement formed a parabola given by the quadratic equation \( h = -16t^2 + 2t + 17 \), where \( h \) is the height in feet and \( t \) is the time in seconds. Find the time \( t \) when the arrow hits the ground. Round to the nearest hundredth.

**Practice**

Solve.

1. \( x(2x - 11) = 0 \)

2. \( \frac{12}{x - 6} = \frac{4}{x} \)

3. **Generalize** Using the path of a ball thrown into the air as an example, describe in mathematical terms each part of the graph the path of the ball creates.

4. **Generalize** What does the graph of a quadratic equation look like when there is no solution? one solution? two solutions?
5. Given that \( y \) varies directly with \( x \), identify the constant of variation such that when \( x = 15 \), \( y = 30 \).

6. **Basketball** Ramero shoots a basketball into the air. The ball’s movement forms a parabola given by the quadratic equation \( h = -16t^2 + 7t + 7 \), where \( h \) is the height in feet and \( t \) is the time in seconds. Find the maximum height of the path the basketball makes and the time \( t \) when the basketball hits the ground. Round to the nearest hundredth.

7. **Multiple Choice** What is the equation of the axis of symmetry of the parabola defined by \( y = \frac{1}{4}(x - 4)^2 + 5 \)?
   - A  \( x = 1 \)
   - B  \( x = 4 \)
   - C  \( x = 5 \)
   - D  \( x = -4 \)

8. Solve \(-7x^2 - 10 = 0\) by graphing.

9. Solve \( \frac{6}{x} = \frac{8}{x + 7} \).

10. A deck of ten cards has 5 red and 5 black cards. Cards are replaced in the deck after each draw. Use an equation to find the probability of drawing a black card twice and rolling a 6 on a number cube.

11. **Geometry** The altitude of the right triangle divides the hypotenuse into segments of lengths \( x \) units and 5 units. To find \( x \), solve the equation \( \frac{x + 5}{6} = \frac{6}{x} \).

12. **Multi-Step** Henry starts working a half-hour before Martha. He can complete the job in 4 hours. Martha can complete the same job in 3 hours.
   a. Let \( t \) represent the total time they work together. In terms of \( t \), how long does Henry work?
   b. Use an equation to find how long they work together to complete the job.
   c. How long does Henry work?

13. Find the quotient of \( \frac{a^2 + 10a - 24}{a - 2} \).

14. Simplify \( \sqrt{49y^5} \).

15. **Profit** An entrepreneur makes $3 profit on each object sold. She would like to make $270 plus or minus $30 total. What is the minimum and maximum number of objects she needs to sell?

16. **Data Analysis** A student knows there will be 4 tests that determine her semester grade. She wants her average to be an 85, plus or minus 5 points. What is the minimum and maximum number of points she needs to earn during the semester?

17. Solve the equation \( |10x| - 3 = 87 \).

18. **Exercise** Tom ran a total of \( \frac{7x}{x^2 + 3x - 18} \) miles in August and \( \frac{2x + 1}{7x + 42} \) miles in September. How many more miles did he run in August?

19. Graph the function \( y = 5x^2 - 10x + 5 \).

20. **Verify** A boundary line is a vertical line. The inequality contains a \( < \) symbol. Which half-plane should be shaded on the graph?
21. **Multiple Choice** Which point does not satisfy the inequality \( x + 2y < 5 \)?
   
   A \((0, 0)\)  
   B \((2, 1)\)  
   C \((3, -4)\)  
   D \((-1, 3)\)

*22. **Ages** A boy is \( b \) years old. His father is 23 years older than the boy. The product of their ages is 50. How old is each person?

*23. **Error Analysis** Two students find the roots of \( 3x^2 - 6x = 24 \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3x^2 - 6x = 24 )</td>
<td>( 3x^2 - 6x = 24 )</td>
</tr>
<tr>
<td>( 3x(x - 2) = 24 )</td>
<td>( 3x^2 - 6x - 24 = 0 )</td>
</tr>
<tr>
<td>( 3x = 0 ) ( x - 2 = 0 )</td>
<td>( 3(x^2 - 2x - 8) = 0 )</td>
</tr>
<tr>
<td>( x = 0 ) ( x = 2 )</td>
<td>( 3(x - 4)(x + 2) = 0 )</td>
</tr>
<tr>
<td>( x - 4 = 0 ) ( x + 2 = 0 )</td>
<td>( x = 4 ) ( x = -2 )</td>
</tr>
</tbody>
</table>

24. Does the graph of \( y + 2x^2 = 12 + x \) open upward or downward?

25. Do the side lengths 18, 80, and 82 form a Pythagorean triple?

26. **Multi-Step** The volume of a prism is \( 3x^3 + 12x^2 + 9x \). What are the possible dimensions of the prism?
   
   a. Factor out common terms.
   
   b. Factor completely.
   
   c. Find the dimensions.

27. **Travel** The Jackson family drove 480 miles on Saturday and 300 miles on Sunday. Their average rate on Sunday was 10 miles per hour less than their rate was on Saturday. Write a simplified expression that represents their total driving time.

28. **Multi-Step** At the carnival, a man says that he will guess your weight within 5 pounds.
   
   a. You weigh 120 pounds. Write an absolute-value inequality to show the range of acceptable guesses.
   
   b. Solve the inequality to find the actual range of acceptable guesses.

29. **Verify** If the numerator of a rational expression is a polynomial and the denominator of the rational expression is a different polynomial, will factoring the polynomials always provide a way to simplify the expression? Verify your answer by giving an example.

30. If a 9% decrease from the original price resulted in a new price of $227,500, what was the original price?
A quadratic function is a function that can be written in the form 
\[ f(x) = ax^2 + bx + c, \]  
where \( a \) is not equal to 0.

In Investigation 6, linear functions were graphed as transformations of the 
parent function \( f(x) = x \). Similarly, you can graph a quadratic function as a 
transformation of the quadratic parent function \( f(x) = x^2 \).

**Parameter Changes**

Complete the table of values for \( f(x) = x^2 \) and graph the quadratic parent 
function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

As is the case for the linear parent function, the quadratic parent function 
\( f(x) = x^2 \) can be written as \( f(x) = ax^2 \), where \( a = 1 \). The graph changes 
when other values are substituted for \( a \).

1. Graph \( y = x^2 \) and \( y = 2x^2 \) on the same set of axes. Compare the 
two graphs.
2. Graph \( y = x^2 \) and \( y = \frac{1}{2}x^2 \) on the same set of axes. Compare the 
two graphs.
3. Graph \( y = x^2 \) and \( y = -x^2 \) on the same set of axes. Compare the 
two graphs.
4. **Generalize** What is the effect of \( a \) on the graph of \( y = ax^2 \)?
5. **Predict** How will the graph of \( f(x) = \frac{2}{3}x^2 \) change in relation to the 
quadradic parent function?
6. **Predict** How will the graph of \( f(x) = -4x^2 \) change in relation to the 
quadradic parent function?

The graph of a function of the form \( f(x) = ax^2 \) always crosses the \( y \)-axis at 
\( (0, 0) \). When \( c \neq 0 \), the graph of the function \( f(x) = ax^2 + c \) does not pass 
through the point \( (0, 0) \).

7. Graph the quadratic parent function and the function \( f(x) = x^2 + 1 \) on 
the same set of axes. Compare the two graphs.
8. Graph the quadratic parent function and the function \( f(x) = x^2 - 2 \) on 
the same set of axes.
9. **Predict** How will the graph of \( f(x) = x^2 + 7 \) compare to the graph of the quadratic parent function?

**Combinations of Parameter Changes**

**Predict** How will each graph compare to the graph of the quadratic parent function? Verify your answer with a graphing calculator.

10. \( f(x) = -x^2 + 2 \)
11. \( f(x) = \frac{1}{2}x^2 - 3 \)

**, Investigation Practice**

Describe how the graph for the given values of \( a \) and \( c \) changes in relation to the graph of the quadratic parent function. Verify your answer with a graphing calculator.

a. \( f(x) = ax^2 + c \) for \( a = 2 \) and \( c = 1 \)
b. \( f(x) = ax^2 + c \) for \( a = -3 \) and \( c = -2 \)
c. \( f(x) = ax^2 + c \) for \( a = \frac{1}{2} \) and \( c = 2 \)
d. \( f(x) = ax^2 + c \) for \( a = -\frac{1}{2} \) and \( c = -1 \)

Write an equation for the transformation described. Then graph the original function and the graph of the transformation on the same set of axes.

e. Shift \( f(x) = 2x^2 - 4 \) up 2 units.
f. Shift \( f(x) = 3x^2 + 5 \) down 4 units and open it downward.